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Higher Separation Axioms via Semi*-open sets

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Abstract: *The purpose of this paper is to introduce new separation axioms semi*-regular, semi*-normal, s*-regular, s**-normal using semi*-open sets and investigate their properties. We also study the relationships among themselves and with known axioms regular, normal, semi-regular and semi-normal.*
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I. INTRODUCTION

Separation axioms are useful in classifying topological spaces. Maheswari and Prasad [8, 9] introduced the notion of s-regular and s-normal spaces using semi-open sets. Dorsett [3, 4] introduced the concept of semi-regular and semi-normal spaces and investigate their properties.

In this paper, we define semi*-regular, semi*-normal, s*-regular and s**-normal spaces using semi*-open sets and investigate their basic properties. We further study the relationships among themselves and with known axioms regular, normal, semi-regular and semi-normal.

II. PRELIMINARIES

Throughout this paper (X, τ) will always denote a topological space on which no separation axioms are assumed, unless explicitly stated. If A is a subset of the space (X, τ), *Cl*(A) and *Int*(A) respectively denote the closure and the interior of A in X .

Definition 2.1[7]**:** A subset A of a topological space (X, τ) is called

(i) generalized closed (briefly g-closed) if *Cl*(A)⊆U whenever A⊆U and U is open in X.

(ii) generalized open (briefly g-open) if X\A is g-closed in X.

Definition 2.2: Let A be a subset of X. Then

(i) generalized closure[5] of A is defined as the intersection of all g-closed sets containing A and is denoted by *Cl**(A).

(ii) generalized interior of A is defined as the union of all g-open subsets of A and is denoted by *Int**(A).

Definition 2.3: A subset A of a topological space (X, τ) is called

(i) semi-open [6] (resp. semi*-open[12]) if A⊆*Cl*(*Int*(A)) (resp. A⊆*Cl**(*Int*(A)).

(ii) semi-closed [1] (resp. semi*-closed[13]) if *Int*(*Cl*(A))⊆A (resp. *Int**(*Cl*(A))⊆A).

The class of all semi*-open (resp. semi*-closed) sets is denoted by S*O(X, τ) (resp. S*C(X,τ)).

The semi*-interior of A is defined as the union of all semi*-open sets of X contained in A. It is denoted by *s*Int*(A).The semi*-closure of A is defined as the intersection of all semi*-closed sets in X containing A. It is denoted by *s*Cl*(A).

Theorem 2.4[13]**:** Let A⊆X and let x∈X. Then x∈*s*Cl*(A) if and only if every semi*-open set in X containing x intersects A.

Theorem 2.5[12]**:** (i) Every open set is semi*-open.

(ii) Every semi*-open set is semi-open.

Definition 2.6: A space X is said to be T_1[17] if for every pair of distinct points x and y in X, there is an open set U containing x but not y and an open set V containing y but not x.

Definition 2.7: A space X is R_0 [16] if every open set contains the closure of each of its points.

Theorem 2.8: (i) X is R_0 if and only if for every closed set F, $Cl(\{x\})\cap F=\phi$, for all $x\in X\setminus F$.

(ii) X is semi*-R_0 if and only if for every semi*-closed set F, $s^*Cl(\{x\})\cap F=\phi$, for all $x\in X\setminus F$.

Definition 2.9: A topological space X is said to be

(i) regular if for every pair consisting of a point x and a closed set B not containing x, there are disjoint open sets U and V in X containing x and B respectively.[17]

(ii) s-regular if for every pair consisting of a point x and a closed set B not containing x, there are disjoint semi-open sets U and V in X containing x and B respectively.[8]

(iii) semi-regular if for every pair consisting of a point x and a semi-closed set B not containing x, there are disjoint semi-open sets U and V in X containing x and B respectively.[3]

Definition 2.10: A topological space X is said to be

(i) normal if for every pair of disjoint closed sets A and B in X, there are disjoint open sets U and V in X containing A and B respectively.[17]

(ii) s-normal if for every pair of disjoint closed sets A and B in X, there are disjoint semi-open sets U and V in X containing A and B respectively.[9]

(iii) semi-normal if for every pair of disjoint semi-closed sets A and B in X, there are disjoint semi-open sets U and V in X containing A and B respectively.[4]

Definition 2.11: A function $f:X\longrightarrow Y$ is said to be

(i) closed [17] if $f(V)$ is closed in Y for every closed set V in X.

(ii) semi*-continuous [14] if $f^{-1}(V)$ is semi*-open in X for every open set V in Y.

(iii) semi*-irresolute [15] if $f^{-1}(V)$ is semi*-open in X for every semi*-open set V in Y.

(iv) contra-semi*-irresolute [15] if $f^{-1}(V)$ is semi*-closed in X for every semi*-open set V in Y.

(v) semi*-open [14] if $f(V)$ is semi*-open in Y for every open set V in X.

(vi) pre-semi*-open [14] if $f(V)$ is semi*-open in Y for every semi*-open set V in X

(vii) contra-pre-semi*-open [14] if $f(V)$ is semi*-closed in Y for every semi*-open set V in X.

(viii) pre-semi*-closed [14] if $f(V)$ is semi*-closed in Y for every semi*-closed set V in X.

Lemma 2.12[10]**:** If A and B are subsets of X such that $A\cap B=\phi$ and A is semi*-open in X, then $A\cap s^*Cl(B)=\phi$.

Theorem 2.13[15]: A function $f:X\longrightarrow Y$ is semi*-irresolute if $f^{-1}(F)$ is semi*-closed in X for every semi*-closed set F in Y.

III. REGULAR SPACES ASSOCIATED WITH SEMI*-OPEN SETS.

In this section we introduce the concepts of semi*-regular and s*-regular spaces. Also we investigate their basic properties and study their relationship with already existing concepts.

Definition 3.1: A space X is said to be ***semi*-regular*** if for every pair consisting of a point x and a semi*-closed set B not containing x, there are disjoint semi*-open sets U and V in X containing x and B respectively.

Theorem 3.2: In a topological space X, the following are equivalent:

(i) X is semi*-regular.

(ii) For every $x\in X$ and every semi*-open set U containing x, there exists a semi*-open set V containing x such that $s^*Cl(V)\subseteq U$.

(iii) For every set A and a semi*-open set B such that $A\cap B\neq\phi$, there exists a semi*-open set U such that $A\cap U\neq\phi$ and $s^*Cl(U)\subseteq B$.

(iv) For every non-empty set A and semi*-closed set B such that $A\cap B=\phi$, there exist disjoint semi*-open sets U and V such that $A\cap U\neq\phi$ and $B\subseteq V$.

Proof: (i)⇒(ii): Let U be a semi*-open set containing x. Then $B=X\backslash U$ is a semi*-closed not containing x. Since X is semi*-regular, there exist disjoint semi*-open sets V and W containing x and B respectively. If $y\in B$, W is a semi*-open set containing y that does not intersect V and hence by Theorem 2.4, y cannot belong to $s^*Cl(V)$. Therefore $s^*Cl(V)$ is disjoint from B. Hence $s^*Cl(V)\subseteq U$

(ii)⇒(iii): Let $A\cap B\neq\phi$ and B be semi*-open. Let $x\in A\cap B$. Then by assumption, there exists a semi*-open set U containing x such that $s^*Cl(U)\subseteq B$. Since $x\in A$, $A\cap U\neq\phi$. This proves (iii).

(iii)⇒(iv): Suppose $A\cap B=\phi$, where A is non-empty and B is semi*-closed. Then $X\backslash B$ is semi*-open and $A\cap(X\backslash B)\neq\phi$. By (iii), there exists a semi*-open set U such that $A\cap U\neq\phi$, and $U\subseteq s^*Cl(U)\subseteq X\backslash B$. Put $V=X\backslash s^*Cl(U)$. Hence V is a semi*-open set containing B such that $U\cap V=U\cap(X\backslash s^*Cl(U))\subseteq U\cap(X\backslash U)=\phi$. This proves (iv).

(iv)⇒(i). Let B be semi*-closed and $x\notin B$. Take $A=\{x\}$. Then $A\cap B=\phi$. By (iv), there exist disjoint semi*-open sets U and V such that $U\cap A\neq\phi$ and $B\subseteq V$. Since $U\cap A\neq\phi$, $x\in U$. This proves that X is semi*-regular.

Theorem 3.3: Let X be a semi*-regular space.

(i) Every semi*-open set in X is a union of semi*-closed sets.

(ii) Every semi*-closed set in X is an intersection of semi*-open sets.

Proof: (i) Suppose X is s*-regular. Let G be a semi*-open set and $x\in G$. Then $F=X\backslash G$ is semi*-closed and $x\notin F$. Since X is semi*-regular, there exist disjoint semi*-open sets U_x and V in X such that $x\in U_x$ and $F\subseteq V$. Since $U_x\cap F\subseteq U_x\cap V=\phi$, we have $U_x\subseteq X\backslash F=G$. Take $V_x=s^*Cl(U_x)$. Then V_x is semi*-closed and by Lemma 2.12, $V_x\cap V=\phi$. Now $F\subseteq V$ implies that $V_x\cap F\subseteq V_x\cap V=\phi$. It follows that $x\in V_x\subseteq X\backslash F=G$. This proves that $G=\cup\{V_x : x\in G\}$. Thus G is a union of semi*-closed sets.

(ii) Follows from (i) and set theoretic properties.

Theorem 3.4: If *f* is a semi*-irresolute and pre-semi*-closed injection of a topological space X into a semi*-regular space Y, then X is semi*-regular.

Proof: Let $x\in X$ and A be a semi*-closed set in X not containing x. Since *f* is pre-semi*-closed, $f(A)$ is a semi*-closed set in Y not containing $f(x)$. Since Y is semi*-regular, there exist disjoint semi*-open sets V_1 and V_2 in Y such that $f(x)\in V_1$ and $f(A)\subseteq V_2$. Since *f* is semi*-irresolute, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are disjoint semi*-open sets in X containing x and A respectively. Hence X is semi*-regular.

Theorem 3.5: If *f* is a semi*-continuous and closed injection of a topological space X into a regular space Y and if every semi*-closed set in X is closed, then X is semi*-regular.

Proof: Let $x \in X$ and A be a semi*-closed set in X not containing x. Then by assumption, A is closed in X. Since f is closed, $f(A)$ is a closed set in Y not containing $f(x)$. Since Y is regular, there exist disjoint open sets V_1 and V_2 in Y such that $f(x) \in V_1$ and $f(A) \subseteq V_2$. Since f is semi*-continuous, $f^{1}(V_1)$ and $f^{1}(V_2)$ are disjoint semi*-open sets in X containing x and A respectively. Hence X is semi*-regular.

Theorem 3.6: If $f : X \longrightarrow Y$ is a semi*-irresolute bijection which is pre-semi*-open and X is semi*-regular. Then Y is also semi*-regular.

Proof: Let $f : X \longrightarrow Y$ be a semi*-irresolute bijection which is semi*-open and X be semi*-regular. Let $y \in Y$ and B be a semi*-closed set in Y not containing y. Since f is semi*-irresolute, by Theorem 2.13 $f^{-1}(B)$ is a semi*-closed set in X not containing $f^{-1}(y)$. Since X is semi*-regular, there exist disjoint semi*-open sets U_1 and U_2 containing $f^{-1}(y)$ and $f^{-1}(B)$ respectively. Since f is pre-semi*-open, $f(U_1)$ and $f(U_2)$ are disjoint semi*-open sets in Y containing y and B respectively. Hence Y is semi*-regular.

Theorem 3.7: If f is a continuous semi*-open bijection of a regular space X into a space Y and if every semi*-closed set in Y is closed, then Y is semi*-regular.

Proof: Let $y \in Y$ and B be a semi*-closed set in Y not containing y. Then by assumption, B is closed in Y. Since f is a continuous bijection, $f^{-1}(B)$ is a closed set in X not containing the point $f^{-1}(y)$. Since X is regular, there exist disjoint open sets U_1 and U_2 in X such that $f^{-1}(y) \in U_1$ and $f^{-1}(B) \subseteq U_2$. Since f is semi*-open, $f(U_1)$ and $f(U_2)$ are disjoint semi*-open sets in Y containing x and B respectively. Hence Y is semi*-regular.

Theorem 3.8: If X is semi*-regular, then it is semi*-R_0.

Proof: Suppose X is semi*-regular. Let U be a semi*-open set and $x \in U$. Take $F = X \setminus U$. Then F is a semi*-closed set not containing x. By semi*-regularity of X, there are disjoint semi*-open sets V and W such that $x \in V$, $F \subseteq W$. If $y \in F$, then W is a semi*-open set containing y that does not intersect V. Therefore $y \notin s^*Cl(\{V\})$ $\Rightarrow$ $y \notin s^*Cl(\{x\})$. That is $s^*Cl(\{x\}) \cap F = \phi$ and hence $s^*Cl(\{x\}) \subseteq X \setminus F = U$. Hence X is semi*-$R_0$.

Definition 3.9: A space X is said to be ***s*-regular*** if for every pair consisting of a point x and a closed set B not containing x, there are disjoint semi*-open sets U and V in X containing x and B respectively.

Theorem 3.10: (i) Every regular space is s*-regular.

(ii) Every s*-regular space is s-regular.

Proof: Suppose X is regular. Let F be a closed set and $x \notin F$. Since X is regular, there exist disjoint open sets U and V containing x and F respectively. Then by Theorem 2.5(i) U and V are semi*-open in X. This implies that X is s*-regular. This proves (i).

Suppose X is s*-regular. Let F be a closed set and $x \notin F$. Since X is s*-regular, there exist disjoint semi*-open sets U and V containing x and F respectively. Then by Theorem 2.5(ii) U and V are semi-open in X. This implies that X is s-regular. This proves (ii).

Remark 3.11: The reverse implications of the statements in the above theorem are not true as shown in the following examples.

Example 3.12: Let X={a, b, c, d} with topology τ={ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}, X}. Clearly (X,τ) is s*-regular but not regular.

Example 3.13: Let X={a, b, c, d} with topology τ={ϕ, {a}, {b}, {a, b}, {a, b, c}, X} Clearly (X,τ) is s-regular not s*-regular.

Theorem 3.14: For a topological space X, the following are equivalent:

(i) X is s*-regular.

(ii) For every $x \in X$ and every open set U containing x, there exists a semi*-open set V containing x such that $s^*Cl(V) \subseteq U$.

(iii) For every set A and an open set B such that $A \cap B \neq \phi$, there exists a semi*-open set U such that $A \cap U \neq \phi$ and $s^*Cl(U) \subseteq B$.

(iv) For every non-empty set A and closed set B such that $A \cap B = \phi$, there exist disjoint semi*-open sets U and V such that $A \cap U \neq \phi$ and $B \subseteq V$.

Proof: (i)⇒(ii): Let U be an open set containing x. Then $B = X \setminus U$ is a closed set not containing x. Since X is s*-regular, there exist disjoint semi*-open sets V and W containing x and B respectively. If $y \in B$, W is a semi*-open set containing y that does not intersect V and hence by Theorem 2.4, y cannot belong to $s^*Cl(V)$. Therefore $s^*Cl(V)$ is disjoint from B. Hence $s^*Cl(V) \subseteq U$.

(ii)⇒(iii): Let $A \cap B \neq \phi$ and B be open. Let $x \in A \cap B$. Then by assumption, there exists a semi*-open set U containing x such that $s^*Cl(U) \subseteq B$. Since $x \in A$, $A \cap U \neq \phi$. This proves (iii).

(iii)⇒(iv): Suppose $A \cap B = \phi$, where A is non-empty and B is closed. Then $X \setminus B$ is open and $A \cap (X \setminus B) \neq \phi$. By (iii), there exists a semi*-open set U such that $A \cap U \neq \phi$, and $U \subseteq s^*Cl(U) \subseteq X \setminus B$. Put $V = X \setminus s^*Cl(U)$. Hence V is a semi*-open set containing B such that $U \cap V = U \cap (X \setminus s^*Cl(U)) \subseteq U \cap (X \setminus U) = \phi$. This proves (iv).

(iv)⇒(i). Let B be closed and $x \notin B$. Take $A = \{x\}$. Then $A \cap B = \phi$. By (iv), there exist disjoint semi*-open sets U and V such that $U \cap A \neq \phi$ and $B \subseteq V$. Since $U \cap A \neq \phi$, $x \in U$. This proves that X is s*-regular.

Theorem 3.15: If X is a regular T_1 space, then for every pair of distinct points of X there exist semi*-open sets containing them whose semi*-closures are disjoint.

Proof: Let x, y be two distinct points in the regular T_1 space X. Since X is T_1, {y} is closed. Since X is regular, there exist disjoint open sets U_1 and U_2 containing x and {y} respectively. By Theorem 3.9(i), X is s*-regular and hence by Theorem 3.13, there exist semi*-open sets V_1 and V_2 containing x, y such that $s^*Cl(V_1) \subseteq U_1$ and $s^*Cl(V_2) \subseteq U_2$. Since U_1 and U_2 are disjoint, $s^*Cl(V_1)$ and $s^*Cl(V_2)$ are disjoint. This proves the theorem.

Theorem 3.16: Every semi*-regular space is s*-regular.

Proof: Suppose X is semi*-regular. Let F be a closed set and $x \notin F$. Then by Theorem (i), F is semi*-closed in X. Since X is semi*-regular, there exist disjoint semi*-open sets U and V containing x and F respectively. This implies that X is s*-regular.

Theorem 3.17: (i) Every s*-regular T_1 space is semi*-T_2.

(ii) Every semi*-regular semi*-T_1 space is semi*-T_2.

Proof: Suppose X is s*-regular and T_1. Let x and y be two distinct points in X. Since X is T_1, {x} is closed and $y \notin \{x\}$. Since X is s*-regular, there exist disjoint semi*-open sets U and V in X containing {x} and y respectively. It follows that X is semi*-T_2. This proves (i).

Suppose X is semi*-regular and semi*-T_1. Let x and y be two distinct points in X. Since X is semi*-T_1, {x} is semi*-closed and $y \notin \{x\}$. Since X is semi*-regular, there exist disjoint semi*-open sets U and V in X containing {x} and y respectively. It follows that X is semi*-T_2. This proves (ii).

Theorem 3.18: Let X be an s*-regular space.

i) Every open set in X is a union of semi*-closed sets.

ii) Every closed set in X is an intersection of semi*-open sets.

Proof: (i) Suppose X is s*-regular. Let G be an open set and $x\in G$. Then $F=X\setminus G$ is closed and $x\notin F$. Since X is s*-regular, there exist disjoint semi*-open sets U_x and U in X such that $x\in U_x$ and $F\subseteq U$. Since $U_x\cap F\subseteq U_x\cap U=\phi$, we have $U_x\subseteq X\setminus F=G$. Take $V_x=s^*Cl(U_x)$. Then V_x is semi*-closed. Now $F\subseteq U$ implies that $V_x\cap F\subseteq V_x\cap U=\phi$. It follows that $x\in V_x\subseteq X\setminus F=G$. This proves that $G=\cup\{V_x : x\in G\}$. Thus G is a union of semi*-closed sets.

(ii) Follows from (i) and set theoretic properties.

IV. NORMAL SPACES ASSOCIATED WITH SEMI*-OPEN SETS.

In this section we introduce variants of normal spaces namely semi*-normal spaces and s**-normal spaces and investigate their basic properties. We also give characterizations for these spaces.

Definition 4.1: A space X is said to be ***semi*-normal*** if for every pair of disjoint semi*-closed sets A and B in X, there are disjoint semi*-open sets U and V in X containing A and B respectively.

Theorem 4.2: In a topological space X, the following are equivalent:

(i) X is semi*-normal.

(ii) For every semi*-closed set A in X and every semi*-open set U containing A, there exists a semi*-open set V containing A such that $s^*Cl(V)\subseteq U$.

(iii) For each pair of disjoint semi*-closed sets A and B in X, there exists a semi*-open set U containing A such that $s^*Cl(U)\cap B=\phi$.

(iv) For each pair of disjoint semi*-closed sets A and B in X, there exist semi*-open sets U and V containing A and B respectively such that $s^*Cl(U)\cap s^*Cl(V)=\phi$.

Proof: (i)⇒(ii): Let U be a semi*-open set containing the semi*-closed set A. Then $B=X\setminus U$ is a semi*-closed set disjoint from A. Since X is semi*-normal, there exist disjoint semi*-open sets V and W containing A and B respectively. Then $s^*Cl(V)$ is disjoint from B, since if $y\in B$, the set W is a semi*-open set containing y disjoint from V. Hence $s^*Cl(V)\subseteq U$.

(ii)⇒(iii): Let A and B be disjoint semi*-closed sets in X. Then $X\setminus B$ is a semi*-open set containing A. By (ii), there exists a semi*-open set U containing A such that $s^*Cl(U)\subseteq X\setminus B$. Hence $s^*Cl(U)\cap B=\phi$. This proves (iii).

(iii)⇒(iv): Let A and B be disjoint semi*-closed sets in X. Then, by (iii), there exists a semi*-open set U containing A such that $s^*Cl(U)\cap B=\phi$. Since $s^*Cl(U)$ is semi*-closed**,** B and $s^*Cl(U)$ are disjoint semi*-closed sets in X. Again by (iii), there exists a semi*-open set V containing B such that $s^*Cl(U)\cap s^*Cl(V)=\phi$. This proves (iv).

(iv)⇒(i): Let A and B be the disjoint semi*-closed sets in X. By (iv), there exist semi*-open sets U and V containing A and B respectively such that $s^*Cl(U)\cap s^*Cl(V)=\phi$. Since $U\cap V\subseteq s^*Cl(U)\cap s^*Cl(V)$, U and V are disjoint semi*-open sets containing A and B respectively. Thus X is semi*-normal.

Theorem 4.3: For a space X, then the following are equivalent:

(i) X is semi*-normal.

(ii) For any two semi*-open sets U and V whose union is X, there exist semi*-closed subsets A of U and B of V whose union is also X.

Proof: (i)⇒(ii): Let U and V be two semi*-open sets in a semi*-normal space X such that $X=U\cup V$. Then X\U, X\V are disjoint semi*-closed sets. Since X is semi*-normal, there exist disjoint semi*-open sets G_1 and G_2 such that $X\backslash U\subseteq G_1$ and $X\backslash V\subseteq G_2$. Let $A=X\backslash G_1$ and $B=X\backslash G_2$. Then A and B are semi*-closed subsets of U and V respectively such that $A\cup B=X$. This proves (ii).

(ii)⇒(i): Let A and B be disjoint semi*-closed sets in X. Then X\A and X\B are semi*-open sets whose union is X. By (ii), there exists semi*-closed sets F_1 and F_2 such that $F_1\subseteq X\backslash A$, $F_2\subseteq X\backslash B$ and $F_1\cup F_2=X$. Then $X\backslash F_1$ and $X\backslash F_2$ are disjoint semi*-open sets containing A and B respectively. Therefore X is semi*-normal.

Definition 4.4: A space X is said to be ***s**-normal*** if for every pair of disjoint closed sets A and B in X, there are disjoint semi*-open sets U and V in X containing A and B respectively.

Theorem 4.5:
(i) Every normal space is s**-normal.
(ii) Every s**-normal space is s-normal.
(iii) Every semi*-normal space is s**-normal.

Proof: Suppose X is normal. Let A and B be disjoint closed sets in X. Since X is normal, there exist disjoint open sets U and V containing A and B respectively. Then by Theorem 2.5(i), U and V are semi*-open in X. This implies that X is s**-normal. This proves (i).

Suppose X is s**-normal. Let A and B be disjoint closed sets in X. Since X is s**-normal, there exist disjoint semi*-open sets U and V containing A and B respectively. Then by Theorem 2.5(ii), U and V are semi-open in X. This implies that X is s-normal. This proves (ii).

Suppose X is semi*-regular. Let A and B be disjoint closed sets in X. Then by Theorem 2.5(i), A and B are disjoint semi*-closed sets in X. Since X is semi*-regular, there exist disjoint semi*-open sets U and V containing A and B respectively. Therefore X is s**-normal. This proves (iii).

Theorem 4.6: In a topological space X, the following are equivalent:

(i) X is s**-normal.

(ii) For every closed set F in X and every open set U containing F, there exists a semi*-open set V containing F such that $s^*Cl(V)\subseteq U$.

(iii) For each pair of disjoint closed sets A and B in X, there exists a semi*-open set U containing A such that $s^*Cl(U)\cap B=\phi$.

Proof: (i)⇒(ii): Let U be a open set containing the closed set F. Then $H=X\backslash U$ is a closed set disjoint from F. Since X is s**-normal, there exist disjoint semi*-open sets V and W containing F and H respectively. Then $s^*Cl(V)$ is disjoint from H, since if $y\in H$, the set W is a semi*-open set containing y disjoint from V. Hence $s^*Cl(V)\subseteq U$.

(ii)⇒(iii): Let A and B be disjoint closed sets in X. Then X\B is an open set containing A. By (ii), there exists a semi*-open set U containing A such that $s^*Cl(U)\subseteq X\backslash B$. Hence $s^*Cl(U)\cap B=\phi$. This proves (iii).

(iii)⇒(i): Let A and B be the disjoint semi*-closed sets in X. By (iii), there exists a semi*-open set U containing A such that $s^*Cl(U)\cap B=\phi$. Take $V=X\backslash s^*Cl(U)$. Then U and V are disjoint semi*-open sets containing A and B respectively. Thus X is s**-normal.

Theorem 4.7: For a space X, then the following are equivalent:

(i) X is s**-normal.

(ii) For any two open sets U and V whose union is X, there exist semi*-closed subsets A of U and B of V whose union is also X.

Proof: (i)⇒(ii): Let U and V be two open sets in an s**-normal space X such that X=U∪V. Then X\U, X\V are disjoint closed sets. Since X is s**-normal, there exist disjoint semi*-open sets G_1 and G_2 such that X\U⊆G_1 and X\V⊆G_2. Let A=X\$G_1$ and B=X\$G_2$. Then A and B are semi*-closed subsets of U and V respectively such that A∪B=X. This proves (ii).

(ii)⇒(i): Let A and B be disjoint closed sets in X. Then X\A and X\B are open sets whose union is X. By (ii), there exists semi*-closed sets F_1 and F_2 such that F_1⊆X\A, F_2⊆X\B and $F_1 \cup F_2$=X. Then X\$F_1$ and X\$F_2$ are disjoint semi*-open sets containing A and B respectively. Therefore X is s**-normal.

Remark 4.8: It is not always true that an s**-normal space X is s*-regular as shown in the following example. However it is true if X is R_0 as seen in Theorem 4.10.

Example 4.9: Let X={a, b, c, d} with topology τ={ϕ, {a}, {b}, {a, b}, {a, b, c}, X}. Clearly (X,τ) is s**-normal but not s*-regular.

Theorem 4.10: Every s**-normal R_0 space is s*-regular.

Proof: Suppose X is s**-normal and R_0. Let F be a closed set and x∉F. Since X is R_0, by Theorem 2.8(i), *Cl*({x})∩F=ϕ. Since X is s**-normal, there exist disjoint semi*-open sets U and V in X containing *Cl*({x}) and F respectively. It follows that X is s*-regular.

Corollary 4.11: Every s**-normal T_1 space is s*-regular.

Proof: Follows from the fact that every T_1 space is R_0 and Theorem 4.10.

Theorem 4.12: If *f* is an injective and semi*-irresolute and pre-semi*-closed mapping of a topological space X into a semi*-normal space Y, then X is semi*-normal.

Proof: Let *f* be an injective and semi*-irresolute and pre-semi*-closed mapping of a topological space X into a semi*-normal space Y. Let A and B be disjoint semi*-closed sets in X. Since *f* is a pre-semi*-closed function, *f*(A) and *f*(B) are disjoint semi*-closed sets in Y. Since Y is semi*-normal, there exist disjoint semi*-open sets V_1 and V_2 in Y containing *f*(A) and *f*(B) respectively. Since *f* is semi*-irresolute, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are disjoint semi*-open sets in X containing A and B respectively. Hence X is semi*-normal.

Theorem 4.13: If *f* is an injective and semi*-continuous and closed mapping of a topological space X into a normal space Y and if every semi*-closed set in X is closed, then X is semi*- normal.

Proof: Let A and B be disjoint semi*-closed sets in X. By assumption, A and B are closed in X.
Then *f*(A) and *f*(B) are disjoint closed sets in Y. Since Y is normal, there exist disjoint open sets V_1 and V_2 in Y such that *f*(A)⊆V_1 and *f*(B)⊆V_2.Then $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are disjoint semi*-open sets in X containing A and B respectively. Hence X is semi*- normal.

Theorem 4.14: If f:X⟶Y is a semi*-irresolute injection which is pre-semi*-open and X is semi*-normal, then Y is alsosemi*-normal.

Proof: Let f:X⟶Y be a semi*-irresolute surjection which is semi*-open and X be semi*-normal. Let A and B be disjoint semi*-closed sets in Y. Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint semi*-closed sets in X. Since X is semi*-normal, there exist disjoint semi*-open sets U_1 and U_2 containing $f^{-1}(A)$ and $f^{-1}(B)$ respectively. Since *f* is pre-semi*-open, $f(U_1)$ and $f(U_2)$ are disjoint semi*-open sets in Y containing A and B respectively. Hence Y is semi*-normal.

Remark 4.15: It is not always true that a semi*-normal space X is semi*-regular as shown in the following example. However it is true if X is semi*-R_0 as seen in Theorem 4.17.

Example 4.16: Let X={a, b, c, d} with topology $\tau=\{\phi, \{a, b\}, X\}$. Clearly (X,τ) is semi*-normal but not semi*-regular.

Theorem 4.17: Every semi*-normal space that is semi*-R_0 is semi*-regular.

Proof: Suppose X is semi*-normal that is semi*-R_0. Let F be a semi*-closed set and $x\notin F$. Since X is semi*-R_0, by Theorem 2.8(ii), $s^*Cl(\{x\})\cap F=\phi$. Since X is semi*-normal, there exist disjoint semi*-open sets U and V in X containing $s^*Cl(\{x\})$ and F respectively. It follows that X is semi*-regular.

Corollary 4.18: Every semi*-normal semi*-T_1 space is semi*-regular.

Proof: Follows from the fact that every semi*-T_1 space is semi*-R_0 and Theorem 4.13.

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