Research Inventy: International Journal Of Engineering And Science
Issn: 2278-4721, Vol.2, Issue 6 (March 2013), Pp 11-14
Www.Researchinventy.Com

A Study Of Convolution Closure Of Idmrl Class Of Life Distribution

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Abstract: *This paper, based on the relation between the survival function and the mean residual life(MRL)function, obtains the sufficient condition of the convolution closure of IDMRL class of life distribution and some relevant counterexamples, and then significantly presents the discovery of the SMC (scalar multiplication closure) of IDMRL class of life distribution, and consequently, in the final part, the Poisson shock model is transformed into a mixture model, as serves more effectively in the analysis of survival and reliability.*
Keywords: *MRL; convolution closure; SMC; Poisson shock model; mixture*

I. Introduction

IDMRL class of life distribution has received continous and increasing attention in the recent years. Anis(2012) investigates the weak convergence of IDMRL distribution; and Anis(2010, 2012), Mitra and Basu(1996), Mitra and Pandey(2011), etc. discuss the relation between IDMRL class of life distributions and Poisson shock model. Chen(2000) studies the new two-parameter lifetime distribution with IDMRL properties. Bebbington, Lai and Zitikis(2007) discover a two parameter flexible Weibull distribution which performs with very complicated monotonicity of failure rate and MRL functions. Besides, Anis(2012) explores the closure of IDMRL class of life distribution based on mixture model and parallel system. The closure of convolution transformation which has been probed into inadequately is of much importance in analysis of survival and reliability. This paper focuses on the closures of convolution transformation of the IDMRL class of life distribution on the basis of the relation between the failure rate and the MRL funcution and obtains some relevant counterexampers, which leads to some important findings.

II. Convolution Closure Of IDMRL Class Of Distribution

Let X be a certain random variable of life distribution with its corresponding common absolutely continuous cumulative distribution function(cdf) $F(t)$, probability density function(pdf) $f(t)$, survival function $\overline{F}(t)=1-F(t)$, failure rate function $r(t)=f(t)/\overline{F}(t)$ and mean residual life function $e_F(t=E[X-t\mid X>t]=\begin{cases}\int_t^\infty \overline{F}(u)du/\overline{F}(t), & \overline{F}(t)>0,\\ 0, & \overline{F}(t)=0.\end{cases}$ And, the definition of the IDMRL class of life distribution is presented as follows:

Denition 2.1 A distribution F is called an Increasing then Decreasing Mean Residual

Life(**IDMRL**) distribution if there exists a change point τ such that

$$e_F(s) \le e_F(t) \text{ for } 0 \le s \le t < \tau,$$

$$e_F(s) \ge e_F(t) \text{ for } \tau \le s \le t < \infty.$$

Theorem 2.1. Assume that $X_i, i = 1,\dots,k$ be some certain i.i.d. random variables of continuous life distribution with expectation $E(X_i) = \mu$. Let $Y = \sum_{i=1}^{k} X_i$. If the failure rates of $X_i, i = 1,\dots,k$ and Y perform a bathtub shape, $\mu \cdot f(0) > 1$ and $k\mu \cdot f_Y(0) > 1$, then the distributions of $X_i, i = 1,\dots,k$ and Y are within the con ne of IDMRL class of life distribution.

Proof: Notice that

$$e_{X_i}(0) \cdot r_{X_i}(0) = E[X_i - 0 \mid X_i > 0] \cdot f(0) = \mu \cdot f(0) > 1, \text{ and}$$

$$e_Y(0) \cdot r_Y(0) = E[Y - 0 \mid Y > 0] \cdot f_Y(0) = k\mu \cdot f_Y(0) > 1,$$ by the statement of Gupta and Lvin(2005), the above results can be easily obtained.

A significant result similar to the theorem above can be inferred as follows:

Theorem 2.2. Assume that X is a certain random variable of continuous life distribution with expectation $E(X) = \mu$. k is a positive integer. If $\mu \cdot f(0) > 1$ and the failure rate of X is in a bathtub shape, then both X and $k \cdot X$ are within the confine of IDMRL class of life distribution.

Proof: Obviously,

$$e_{kX}(0) \cdot r_{kX}(0) = E[kX - 0 \mid kX > 0] \cdot f_{kX}(0) = k\mu \cdot \left\{\tfrac{1}{k} f(\tfrac{t}{k})\right\}_{t=0} = \mu \cdot f(0) > 1.$$

Meanwhile, $r_{kX}(t) = \frac{1}{k} r_X(\frac{t}{k})$.

This means that the failure rate functions rX(t) and rkX(t) perform the same bathtub shape.

So, both X and $k \cdot X$ are within the confine of IDMRL class of life distribution.

But in the general case, the closure of IDMRL class does not exist. Then, a type of counterexamples are demonstrated as follows:

Counterexample: Take an example of Poisson shock model$\left(\overline{H}(t) = \sum_{0}^{\infty} \frac{(\lambda t)^k}{k!} e^{-\lambda t} \overline{P}_k\right)$: $\overline{P}_0 = 1$ $\overline{P}_1 = 1/4$, $\overline{P}_2 = \overline{P}_3 = 1/20$, $\overline{P}_k = 0, k \ge 4$ 4(Belzunce, Ortega and Ruiz, 2007).

Obviously, the corresponding distribution is within the confine of IDMRL class of life distribution with the change point $\tau = 2$.

Assume that $X_i \sim H(\lambda_i), i = 1,2,\ \lambda_1 = 0.3, \lambda_2 = 0.9.$ Let $Y = X_1 + X_2$, the density function of Y is

$$f_Y(t) = 0.3506e^{-0.3t} - 0.3506e^{-0.9t} + 0.0240te^{-0.3t} + 0.0175te^{-0.9t}$$

$$-0.0010t^2e^{-0.3t} - 0.0072t^2e^{-0.9t} + 0.0001t^3e^{-0.3t} - 0.0018t^3e^{-0.9t}.$$

The corresponding survival function

$$\overline{F}_Y(t) = 1.4512e^{-0.3t} - 0.4037e^{-0.9t} + 0.0848te^{-0.3t} + 0.0128te^{-0.9t}$$

$$+0.0007t^2e^{-0.3t} - 0.0145t^2e^{-0.9t} + 0.0004t^3e^{-0.3t} - 0.0020t^3e^{-0.9t}.$$

The associated mean residual life function

$$e_Y(t) = [6.1359e^{-0.3t} - 0.5220e^{-0.9t} + 0.3895te^{-0.3t} - 0.0661te^{-0.9t}$$

$$+0.0160t^2e^{-0.3t} - 0.0233t^2e^{-0.9t} + 0.0014t^3e^{-0.3t} - 0.0022t^3e^{-0.9t}]/\overline{F}_Y(t).$$

It is obvious that the distribution of random variable Y is not within the confine of IDMRL class of life distribution. The comparison of the mean residual life function of X_1, X_2, Y are demonstrated in the following Figure 1(I, II, III):

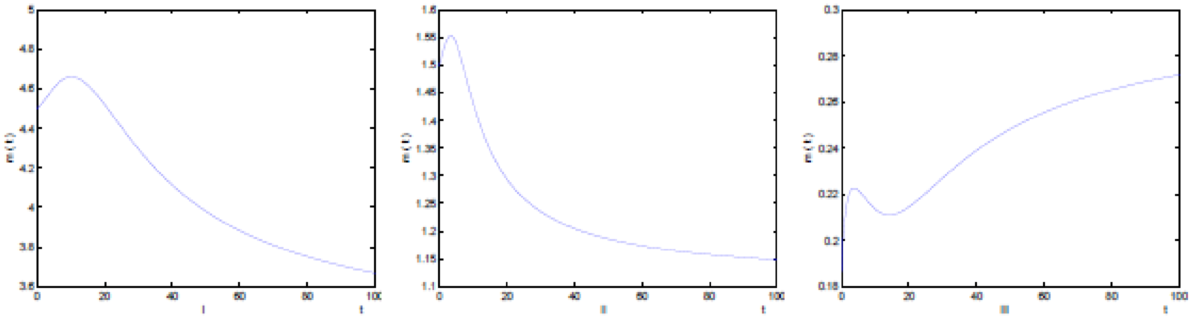

Figure 1: The plots of MRL of X_1, X_2, Y.

Indeed, there are a large number of λ values which fulfill this condition.

In fact, Poisson shock model can be viewed as a mixture. Notice that

$$h(t) = -\frac{d}{dt}\{\overline{H}(t)\} = \sum_{k=0}^{\infty} \frac{\lambda^{k+1}t^k}{k!}e^{-\lambda t}\left(\overline{P}_k - \overline{P}_{k+1}\right),$$ Denote $f_k(t) = \frac{\lambda^{k+1}t^k}{k!}e^{-\lambda t}$, $p_k = \overline{P}_k - \overline{P}_{k+1}$.

Then $p_k \geq 0$, $\sum_{k=0}^{\infty} p_k = 1$ and $h(t) = \sum_{k=0}^{\infty} p_k f_k(t)$.

Meanwhile, many achievements have been made in the study of the mixture model which can be employed for further research on IDMRL class of life distribution.

III. Discussion

There is no doubt that the failure rate and the mean residual life of random variables, especially for IDMRL class of life distribution, become more complicated in the condition of convolution transformation. At present, the function structures employed for the study IDMRL class mainly include mixture model, piecewise function, flexible Weibull distribution(Bebbington, Lai and Zitikis, 2007), and a new two-parameter lifetime distribution(developed by Chen(2000)), etc. However, these function structures usually mean mechanical and ineffective calculation, which suggests the necessity of exploration of a simpler and more effective model.

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