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Convective Flow of Two Immiscible Fluids and Heat Transfer With Porous Along an Inclined Channel with Pressure Gradient.

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Abstract: *This paper presents a novel of Two Immiscible fluids flow and heat transfer with porous along an inclined channel with pressure gradient. Two stationary plates held at constant different temperatures with one region filled with porous material saturated with a viscous fluid and another region with a clear viscous fluid. The differential equation is solved analytically. We found that the presence of porous structure, material constants and different liquid can effectively control fluid flow. Results for a wide range of governing parameters such as Pressure parameter, Grashof number, angle of inclination and ratios of heights of the two layersare plotted on the velocity and temperature fields. The results obtained through this method are found to be in excellent agreement with earlier results obtained analytically.*

Keywords: *Porous medium, convective flow, inclined channel*

Nomenclature

$c_p$specific heat at constant pressure

$g$acceleration due to gravity

$h$ratio of the heights of the region $\left(\frac{h_2}{h_1}\right)$

$h_1$height of the region –I

$h_2$height of the region - II

G_rGrashof number $\frac{g\beta_1 h_1^3 (T-T_{w2})}{\vartheta_1^2}$

ReReynolds number $\left(\frac{u_0 h_1}{\vartheta_1}\right)$

$k$permeability of the porous medium

Kratio of the thermal conductivities $\left(\frac{k_1}{k_2}\right)$

$k_1$effective thermal conductivity of the fluid saturated porous medium in region- I

$k_2$thermal conductivity of the fluid in region-II

$m$ratio of the viscosities $\left(\frac{\mu_1}{\mu_2}\right)$

$p$pressure

Pnondimensional pressure gradient $\left(-\frac{h_1^2}{\mu_1 u_0}\frac{dp}{dx}\right)$

PrPrandtl number $\left(\frac{\mu_1 c_p}{k_1}\right)$

Ttemperature

T_{w1}, T_{w2} temperature of the boundaries

$u$velocity

x, y, zspace coordinates

Greek Symbols

βratio of the coefficients of the thermal expansion $\left(\frac{\beta_2}{\beta_1}\right)$

σporous parameter $\left(\frac{h_1}{\sqrt{k}}\right)$

ρdensity of the fluid

ϑkinematic viscosity

μviscosity

θnondimensional temperature $\left(\frac{T-T_w}{\frac{u_0^2\vartheta_1}{k_1}}\right)$

Subscripts
1 & 2 refers to the quantities for region I and II respectively
$w$wall conditions

I Introduction

Convective heat transfer and fluid flow in a system containing simultaneously a reservoir and a porous medium saturated with fluid is of great mathematical and physical interest. More specifically the existence of a fluid layer adjacent to a layer of fluid saturated porous medium is a common occurrence in both geophysical and engineering environments. Composite systems are part of numerous and other engineering applications also, such as fibrous and granular insulation, porous insulation of ducts, ambient air heat transfer from heir covered skin, grain storage, and drying paper. Freezing of soila and melting the ice frozen soils due to the change in weather conditions also require the knowledge of interaction mechanism between the fluid and porous layers. Two – fluid flow and heat transfer in an inclined channel containing porous and fluid layers was studied analytically by Malashetty et al [1]. Umavathi et al [2,3,4,5] analyzed steady and unsteady flow and heat transfer of immiscible fluids in a horizontal channel. An extensive review of convective flow and heat transfer between fluid and porous layers has been done by Prasad [6]. Srinivasan and Vafai [7] have reported a theoretical study on two immiscible fluid systems in a porous medium, taking into account the non-Darcian boundary and inertia effects. Chamkha [8] analyzed the flow of two immiscible fluids in porous and non-porous channels. Masuoka [9] has observed convective flow in a layer of fluid heated from below and divided by a horizontal porous wall suppresses the convection. Recently, heat transfer in channels partially filled with porous media has received considerable attention and was the focus of several investigations Chikh et al [10] and Vafai and Kim [11]In the present work, we study the convective flow of two immiscible fluids and heat transfer with porous along an inclined channel with pressure gradient, containing porous layer saturated with a fluid and a clear viscous fluid layer. Processes involving heat and mass transfer are often encountered in the chemical industry, in reservoir engineering connections, with thermal recovery processes, and in the study of the dynamics of salty hot springs in the sea.

II Mathematical Analysis

We considered coupling of governing equations for the fluid region with the equations for the porous region with the equations for the porous region through an appropriate set of matching conditions at the fluid/porous medium interface, we assume the continuity of velocity, shear stress, temperature and heat flux at the interface. The region $-h_2 \leq y \leq 0$ is occupied by clear fluid. The two walls of the channel are held at different temperature $T_{w1} > T_{w2}$ and are inclined, making an angle $\emptyset$ with the horizontal. In the analysis, the porous medium is considered to be homogeneous and isotropic. The fluid within the porous medium saturates the solid matrix and both are in local thermodynamic equilibrium. The fluids in the two regions are different with different properties. The flow is assumed to be steady, laminar and incompressible. Further, the fluid on both regions is assumed to be driven by a common constant pressure gradient $-\frac{\partial p}{\partial x}$ and that existence of heat transfer does not affect the pressure gradient with the assumptions mentioned aboved, the governing equations of motion and energy can be written as follows:

Region I

$$V_0\frac{du_1'}{dy'} = \mu_1\frac{d^2u_1'}{dy'^2} + g\beta_1(T_1 - T_{w2})sin\emptyset - \frac{\partial p'}{\partial x'} \qquad (1)$$

$$V_0 c_p\frac{dT_1'}{dy'} = \frac{k_1}{\vartheta_1}\frac{d^2T_1'}{dy^2} + \left(\frac{du_1}{dy}\right)^2 + \frac{u_1^2}{k} \qquad (2)$$

Region II

$$V_0\frac{du_2'}{dy'} = \mu_2\frac{d^2u_2'}{dy'^2} + g\beta_2(T_2 - T_{w2})sin\emptyset - \frac{\partial p'}{\partial x'} \qquad (3)$$

$$V_0 c_p\frac{dT_2'}{dy'} = \frac{k_2}{\vartheta_2}\frac{d^2T_2'}{dy^2} + \left(\frac{du_2}{dy}\right)^2 + \frac{u_2^2}{k} \qquad (4)$$

The appropriate boundary and matching conditions for the problem under consideration can be written as
$u_1(h_1) = 0$
$u_1(0) = u_2(0)$

$$u_2(-h_2) = 0$$
$$\mu_1 \frac{du_1}{dy} = \mu_2 \frac{du_2}{dy} \text{at} y = 0 \qquad (5)$$
$$T_1(h_1) = T_{w1}$$
$$T_1(0) = T_2(0)$$

$$T_2(-h_2) = T_{w2}$$
$$k_1 \frac{dT_1}{dy} = k_2 \frac{dT_2}{dy} \text{at} y = 0 \qquad (6)$$

In order to non-dimensionalise the governing equations the following transformations are used

$$y = \frac{y'}{h}, \quad u = \frac{u'}{u_0}, \quad P = -\frac{h_1^2}{\vartheta_1 u_0} \frac{dp'}{dx'}, \quad Re = \frac{u_0 h_1}{\vartheta_1}, \quad s = \frac{V_0 h_1}{\vartheta_1}, \quad Pr = \frac{\mu_1 c_p}{k_1}, \quad \sigma = \frac{h_1}{\sqrt{k}}$$
$$Gr = \frac{g\beta_1 h_1^3 (T - T_{w2})}{\vartheta_1^2}, \quad \theta = \left(\frac{T - T_w}{\frac{u_0^2 \vartheta_1}{k_1}} \right), \quad \delta = \frac{\vartheta_1}{h_2} \qquad (7)$$

The ratios of different parameters that appears in governing equations are

$$\beta = \frac{\beta_2}{\beta_1}, \quad h = \frac{h_2}{h_1}, \quad k = \frac{k_1}{k_2}, \quad m = \frac{\mu_1}{\mu_2}$$

Substitution of the above nondimensional quantities in equations (1)-(4), results in the following equations.

Region-I

$$s \frac{du_1}{dy} = \frac{d^2 u_1}{dy^2} - \frac{Gr sin\emptyset}{Re} + P \qquad (8)$$
$$sPr\delta \frac{d\theta_1}{dy} = \frac{d^2\theta_1}{dy^2} + \left(\frac{du_1}{dy}\right)^2 + \sigma^2 u_1^2 \qquad (9)$$

Region-II

$$shm \frac{du_2}{dy} = \frac{d^2 u_2}{dy^2} - \frac{Gr sin\emptyset h^2 m\beta}{Re} + Ph^2 m \qquad (10)$$
$$\frac{shPr}{\Gamma} \frac{d\theta_2}{dy} = \frac{d^2\theta_2}{dy^2} + \left(\frac{du_2}{dy}\right)^2 \qquad (9)$$

The transformed boundary and matching conditions are given by

$$u_1(1) = 0$$
$$u_1(0) = u_2(0)$$
$$u_2(-1) = 0$$
$$\frac{du_1}{dy} = \frac{1}{mh} \frac{du_2}{dy} \text{at} y = 0$$
$$\theta_1(1) = 1$$
$$\theta_1(0) = \theta_2(0)$$
$$\theta_2(-1) = 0$$
$$\frac{d\theta_1}{dy} = \frac{1}{kh} \frac{d\theta_2}{dy} \text{at} y = 0$$

Solving the differential equations (8), (9), (10) and (11) satisfying the boundary and interface conditions (12) and (13) we obtain

$$u_1(y) = \frac{1}{s}\left(P - \frac{Gr sin\emptyset}{Re}\right)\left(y + \frac{1}{s}\right) - \frac{c_1}{s} + c_2 \exp(sy) \qquad (14)$$
$$\theta_1(y) = A_1 \exp(sy) + A_2 y\exp(sy) + A_3 \exp(2sy) + A_4 y^3 + A_5 y^2 + A_6 y + A_7 + A_8 c_5 + c_6 \exp(sPr\delta y) \qquad (15)$$
$$u_2(y) = \frac{1}{shm}\left(Ph^2 m - \frac{Gr sin\emptyset h^2 m\beta}{Re}\right)\left(y + \frac{1}{shm}\right) - \frac{c_3}{shm} + c_4 \exp(shmy) \qquad (16)$$
$$\theta_2(y) = B_1 y + B_2 + B_3 \exp(shmy) + B_4 \exp(2shmy) + B_5 c_7 + c_8 \exp\left(\frac{shPr}{\Gamma} y\right) \qquad (17)$$

All the constants defined in the above equations are shown in the appendix

III Results and Discussion

In the previous section, an analytical solution for the problem flow of two immiscible fluids through inclined channel was obtained. Equation (14)-(17) are evaluated numerically, fixing some of the parameters $m = 10,\ Re = 1,\ s = 10,\ \beta = 1,\ \sigma = 1,\ Pr = 0.71,\ \delta = 1,\ \gamma = 1,\ k = 1$ and varying the parameters $P,\ Gr,\ \emptyset,\ h$. In this section, a representative set of results has been selected for presentation in graphical form. We have written a MATLAB programme to compute and generate the graphs for the velocity and temperatures. Some representative results are presented in the form of line graphs in fig. 1-8 to interpret the effects of these parameters. The effect of Pressure parameter P on velocity and temperature profiles is shown in fig. 1 and 2. For fixed porous parameter, the pressure parameter increases on Region-II as both velocity and temperature increases. Its increases more pronounced on temperature. The frictional drag resistance against the convective is very large and as a result, the velocity is very small in porous region while on the temperature it is more significant. The porous parameter σ decreases the fluid velocity thereby increases temperature. The effect of Pressure parameter is more on the temperature profile. In fig.3 and 4, we illustrate how the Grashofnumber Gr affects the velocity and temperature distributions in the inclined channel for fixed values of parameter mention above and varyingGr. The explanation offered for fig. 5 and 6 displays the velocity and temperature profiles in the inclined channel for varying angle of inclination $\emptyset$ with fixed parameter. As we can see, the effect of increasing $\emptyset$ increases the velocity. The driving force increases with the inclination angle. In temperature profile shown on fig.6; as temperature increases $\emptyset$ oscillate which is more pronounced on region II toward to plate 1
In fig.7 and 8, we illustrate the effects of ratios of the heights on velocity and temperature. We observe that the velocity and temperature increases with increases in the values of the ratio of the heights. On the temperature profile, in region-I is flat and region-II convective heat transfer increases more toward to plate 1.

IV Conclusions

The problem of laminar flow in an inclined channel with an immiscible fluid was analyzed. The basic equations governing the flow were solved analytically. The effect of Pressure and other parameters on the flow was promoted for a clear fluid condition and suppressed for porous condition. Both the velocity and temperature increases on clear fluid condition. The convective heat transfer increases at the right wall and decreased at the left wall for porous region. The angle of inclination oscillated on both velocity and temperature in the inclined channel. The rate of heat transfer decreased at the left wall and increased at the right wall for both velocity and temperature for different parameters.

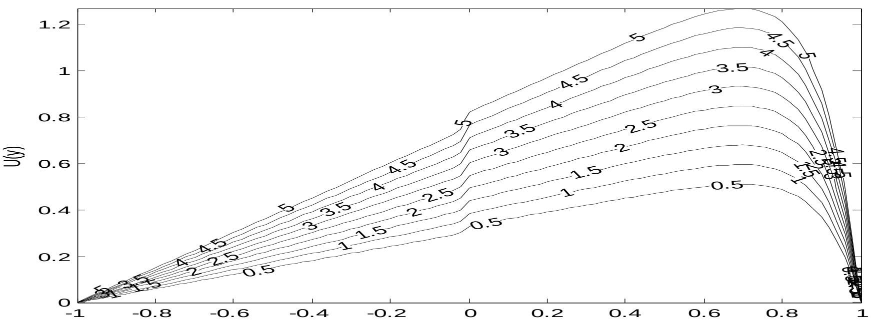

Fig1. Velocity profiles showing effect of Pressure Parameter$\mathrm{P}$

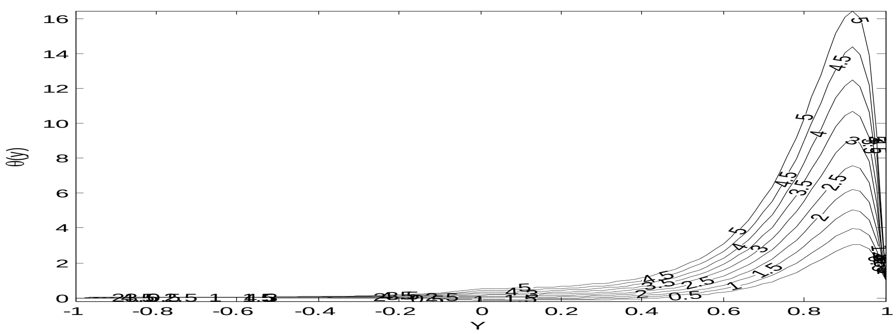

Fig2. Temperature profiles showing effect of Pressure Parameter$\mathrm{P}$

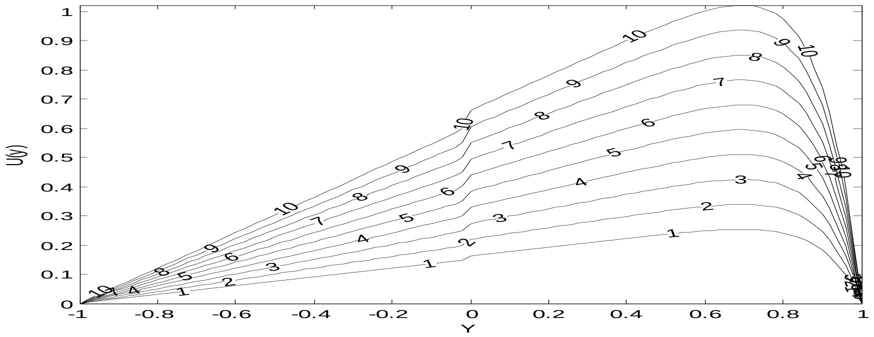

Fig3. Velocity profiles showing effect of Grashof numberGr

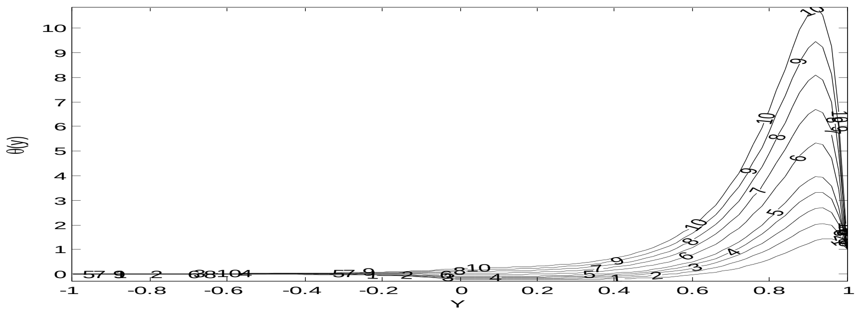

Fig4. Temperature profiles showing effect of Grashof numberGr

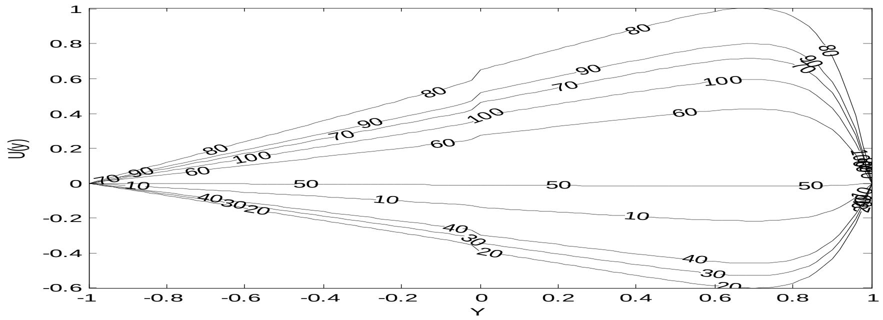

Fig5 Velocity profiles showing effect of Angle of inclination $\emptyset$

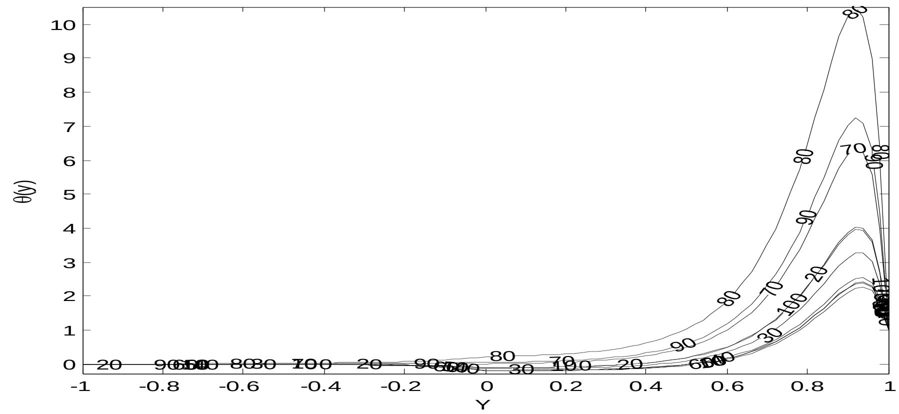

Fig6. Temperature profiles showing effect of Angle of inclination $\emptyset$

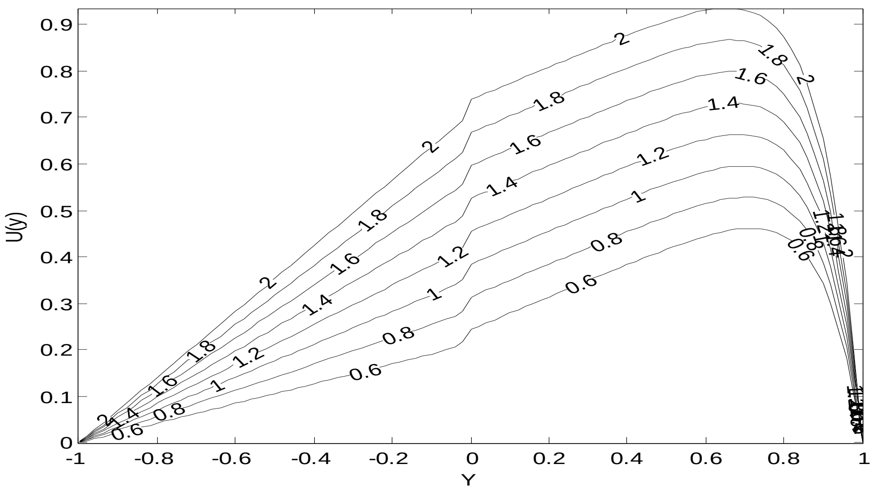

Fig7. Velocity profiles showing effect of Ratio of heights h

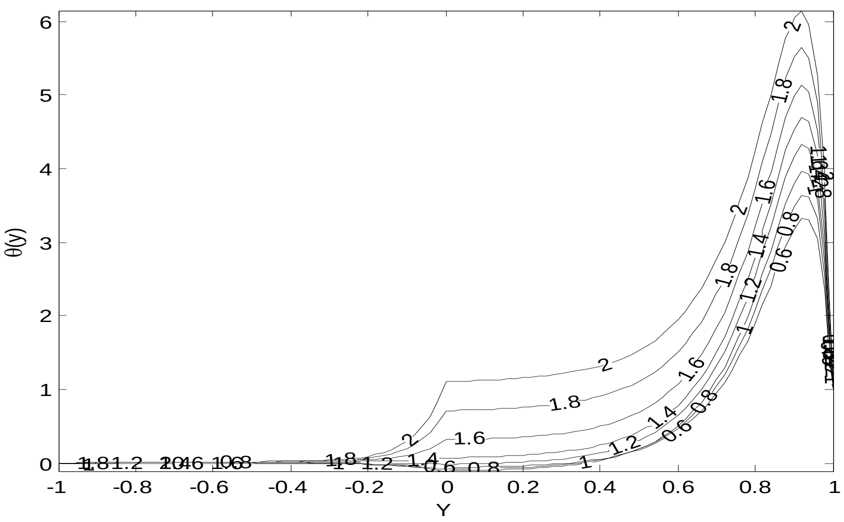

Fig8. Temperature profiles showing effect of Ratio of heights h

Appendix

$N_1 = \left(P - \frac{Grsin\emptyset}{Re}\right)\left(1 + \frac{1}{s}\right)$, $N_2 = sexp(s)$, $N_3 = \left(Ph^2m - \frac{Grsin\emptyset\, h^2 m\beta}{Re}\right)\left(1 - \frac{1}{shm}\right)$

$N_4 = shmexp(-shm)$, $N_5 = \frac{1}{shm}$, $N_6 = \frac{1}{(shm)^2}\left(Ph^2m - \frac{Grsin\emptyset h^2 m\beta}{Re}\right)$, $N_7 = \frac{1}{s}$

$N_8 = \frac{1}{s^2}\left(P - \frac{Grsin\emptyset}{Re}\right)$, $N_9 = \frac{1}{s}\left(P - \frac{Grsin\emptyset}{Re}\right)$, $N_{10} = \frac{1}{shm}\left(Ph^2m - \frac{Grsin\emptyset\, h^2 m\beta}{Re}\right)$

$N_{11} = N_6 - N_8 + N_3N_5 + N_1N_7$, $N_{12} = (1 - N_2N_7)[N_9mh - N_{10}] + N_{11}mh$, $N_{13} = mh(1 - N_4N_5)$

$N_{14} = shm[1 - N_2N_7]$, $c_4 = \frac{N_{12}}{N_{14} - N_{13}}$, $c_2 = \frac{c_4[1 - N_4N_5] + N_{11}}{[1 - N_2N_7]}$, $c_3 = c_4N_4 - N_3$

$c_1 = N_1 + c_2N_2$, $N_{15} = \left(P - \frac{Grsin\emptyset}{Re}\right)$, $N_{16} = \frac{1}{s}\left(P - \frac{Grsin\emptyset}{Re}\right)$, $N_{17} = N_{16}{}^2$, $N_{18} = 2c_2N_{15}$

$N_{19} = s^2c_2^2$, $N_{20} = (\sigma N_{16})^2$, $N_{21} = \frac{N_{20}}{s}$, $N_{22} = 2\sigma^2\left(\frac{N_{16}c_1}{s}\right)$, $N_{23} = 2N_{16}c_2\sigma^2$,

$N_{24} = \frac{2N_{14}c_1\sigma^2}{s^2}$, $N_{25} = \frac{N_{23}}{s}$, $N_{26} = \left(\frac{\sigma c_1}{s}\right)^2$, $N_{27} = 2\sigma^2\left(\frac{c_1c_2}{s}\right)$, $N_{28} = (\sigma c_2)^2$

$N_{29} = sPr\delta$, $N_{30} = N_{17} + N_{21} - N_{24} + N_{26}$, $N_{31} = N_{18} + N_{25} - N_{27} - N_{28}$, $N_{32} = N_{21} - N_{22}$

$$N_{33}=\frac{N_{23}}{s^2}-\frac{N_{31}}{s},\quad N_{34}=-\frac{N_{19}}{2s},\quad N_{35}=-\frac{N_{23}}{s},\quad N_{36}=-\frac{N_{20}}{3},\quad N_{37}=-\frac{N_{32}}{2},\quad N_{38}=-N_{30}$$

$$N_{39}=\frac{N_{33}}{s(1-N_{29})},\quad N_{40}=\frac{N_{34}}{2s(1-N_{29})},\quad N_{41}=\frac{N_{35}}{s(1-N_{29})},\quad N_{42}=-\frac{N_{35}}{(s[1-N_{29}])^2}$$

$$N_{43}=-\frac{N_{36}}{N_{29}},\quad N_{44}=-\frac{3N_{36}}{(N_{29})^2},\quad N_{45}=-\frac{6N_{36}}{{N_{29}}^3},\quad N_{47}=-\frac{N_{37}}{N_{29}},\quad N_{48}=-\frac{2}{{N_{29}}^2},\quad N_{49}=-\frac{2}{{N_{29}}^3}$$

$$N_{50}=-\frac{N_{38}}{N_{29}},\quad N_{51}=-\frac{N_{38}}{{N_{29}}^2},\quad N_{52}=-\frac{1}{N_{29}},$$

$$A_1=N_{39}+N_{42},\quad A_2=N_{41},\quad A_3=N_{40},\quad A_4=N_{43},\quad A_5=N_{44}+N_{47},\quad A_6=N_{45}+N_{48}+N_{50}$$

$$A_7=N_{46}+N_{49}+N_{51},,\quad A_8=N_{52},\quad N_{53}=\frac{1}{shm},\quad N_{54}=Ph^2m-\frac{Grsin\emptyset h^2m\beta}{Re},$$

$$N_{55}=\frac{1}{shm}\left(Ph^2m-\frac{Grsin\emptyset\, h^2m\beta}{Re}\right),\quad N_{56}=\frac{sPrh}{\Gamma},\quad N_{57}=-{N_{55}}^2,\quad N_{58}=-\frac{2c_4N_{53}N_{55}}{shm},$$

$$N_{59}=\frac{c_4^2(N_{53})^2}{2shm},\quad B_1=-\frac{N_{57}}{N_{56}},\quad B_2=-\frac{N_{57}}{{N_{56}}^2},\quad B_3=\frac{N_{58}}{shm-N_{56}},\quad B_4=\frac{N_{56}}{2shm-N_{56}}$$

$$B_5=-\frac{1}{N_{56}},\quad N_{60}=(A_1+A_2)\exp(s)+A_3\exp(2s)+A_4+A_5+A_6+A_7$$

$$N_{61}=B_1-B_2-B_3\exp(-shm)-B_4\exp(-2shm),\quad N_{62}=A_1+A_3+A_7,\quad N_{63}=B_2+B_3+B_4$$

$$N_{64}=B_1+\frac{B_3}{shm}+\frac{B_4}{2shm},\quad N_{65}=\frac{A_1}{s}+A_2+\frac{A_3}{2s}+A_6$$

$$N_{66}=1+N_{62}-N_{60}-N_{63}-N_{61},\quad N_{67}=1-\exp(N_{29}),\quad N_{68}=1-\exp(-N_{56}),\quad N_{69}=N_{56}N_{65}kh-N_{64}N_{56}$$

$$N_{70}=\frac{khN_{56}}{N_{29}},\quad N_{71}=N_{69}N_{68}-N_{66},\quad c_6=\frac{N_{71}}{N_{72}},\quad c_8=\frac{N_{66}+c_6N_{67}}{N_{68}},\quad c_7=\frac{N_{61}-c_8\exp(-N_{56})}{B_5}$$

$$c_5=-\frac{N_{60}+c_6\exp(N_{29})-1}{A_8}$$

V Reference

[1] Malashetty, M.S., Umavathi, J.C. and Prathap Kumar, J., Two fluid flow and heat transfer in an inclined channel containingporous and fluid layer, Heat and Mass Transfer, 40, 871-876. 2004.

[2] Umavathi, J.C., Malashetty, M.S. and Abdul Mateen, "Fully Developed Flow and Heat Transfer in a Horizontal Channel Containing Electrically Conducting Fluid Sandwiched between Two Fluid Layers", Int. J. Appl. MechanicalEngineering, 9, 781-794, 2004.

[3] Umavathi, J.C., Chamkha, A.J., Abdul Mateen and Al-Mudhaf, A., "Unsteady Two Fluid Flow and Heat Transfer in a Horizontal Channel", Heat Mass Transfer, 42, 81-90, 2005.

[4] Umavathi, J.C., Chamkha, A.J., Manjula, M.H. and Al-Mudhaf, A., "Flow and Heat Transfer of a Couple-Stress Fluid Sandwiched between Viscous Fluid Layers", Canadian J. of Physics, 83, 705-720, 2005.

[5] Umavathi, J.C., Prathap Kumar, J. and Chamkha, A.J., "Flow and Heat Transfer of a Micropolar Fluid Sandwiched between Viscous Fluid Layers", Canadian J. of Physics, 86, 961-973, 2008.

[6] Prasad, V., "Convection Flow Interaction and Heat Transfer between Fluid and Porous Layers", Proc. NATO Advanced Study Institute on Convective Heat and Mass Transfer in Porous Media, Turkey, 1990.

[7] Srinivasan, V. and Vafai, K., "Analysis of Linear Encroachment in Two-Immiscible Fluid Systems", ASME J. of Fluid Engineering, 116, 135-139, 1994.

[8] Chamkha, A.J.,"Flow of Two-Immiscible Fluids in Porous and Non-Porous Channels", ASME J. of Fluid Engineering,122, 117-124, 2000.

[9] Masuoka, T.,. Convective currents in a horizontal layer divided by a permeable wall, Bull. Japan, Soc. Mech. Eng., Vol. 17, pp. 225-237. 1974

[10] Chikh, S., Boumedian, A., Bouhadef, K. and Lauriat, G., Analytical solution of non-Darcian forced convection in an annular duct partially filled with porous medium, Int. J. Heat Mass Transfer, Vol. 38, pp. 1543-1551. 1995.

[11] Kim, S.J. and Choi, C.Y.,. Convection heat transfer in porous and overlying layers heated from below, Int. J. Heat MassTransfer, Vol. 39, pp. 319-329. 1996

[12] Kuznetsov, A.V.,. Fluid mechanics and heat transfer in the interface region between a porous medium and a fluid layer: a Boundary layer solution, J. Porous Media, 2(3), 309-321. 1999