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Computational Marvels

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Abstract*: In this paper we will consider several instances where results of computation are contrary to people's expectations. Better understanding of Mathematics helps people to determine the merit of the scheme or the bet they were entering in. Also we will consider examples where knowledge of Mathematics helps in managing money. Some examples are real life examples, some are fables. But all the examples are amusing and enlightening.*

I. Introduction:

G.H. Hardy the pure Mathematician from Cambridge has said that he is proud that his Mathematics has no real life applications. It is true that the motivation for new mathematical ideas is internal, arising from within the subject and rarely from real life situations. But even then Mathematics always finds applications in other sciences. It can be also misused to take advantage of people's ignorance. Ignorance of the people is the reason for success of many bets and investment schemes.

II. Birthday match

In his autobiography (which he calls automathography) famous mathematician Paul Holmos gives the following idea to make money.

In any group of 23 or more people you can make a bet that there will be at least two people in the group with the same birthday. Since there are 366 possible birthdays this seems less likely and there may be people willing to accept your bet. But the probability of a pair having different birthday is 364/365 if we do not think of leap years. Now there are 253 pairs of people in a group of 23 because 23x22/2 = 253.
Each pair having different birthday has the probability $(364/365)^{253} = 0.4993$. Hence the probability of at least one pair having the same birthday is 1 – 0.4993 = 0.5007.

Thus the probability of 2 people having the same birthday in a group of 23 people is more than ½ and your chances of winning improve if the group is bigger. In a group of 57 people it is 0.99, almost a sure win. Indeed, this was tasted in the football world cup 4 years ago. Two teams of 11 players each with a referee make a group of 23. In 7 out of the 10 matches they found birthday match.

III. Power of a geometric series

This legend is similar to the famous chess-board legend.
A clever person approached a millionaire on the first day of a month and made an offer. I will pay you Rs. 1 corer every day and in return you pay me Rs.1 on the first day, Rs. 2 on the second day, Rs. 4 on the third and so on. You must pay double the amount paid on the previous day in return for Rs. one corer. We will continue this way for a month. The millionaire agreed happily without realizing the fast growth of a geometric series. He wished the month had 31 days so that he can benefit more.

The agreement was honored by both. In the end on the 30^{th} day the rich man got Rupees 30 corers but what he paid was more than 3 times the sum. The sum he paid equals Rupees 107,37,41,800. In fact, the rich man looses most of his money on last few days. In last two days itself he ends up paying more than 80 corers. If the month was February without a leap year the rich man would have made a small profit.

IV. Bicycle swindle

In pre-revolutionary Russia there were firms which resorted to an ingenious way of disposing of substandard goods. Such schemes were run in India also.
For example a bicycle worth Rs. 500 is advertised to be sold at Rs.100. The condition is that the buyer gets 4 others to join and they pay Rs 100 each. Thus the buyer actually gets the bicycle for Rs.100 and the seller gets his amount of Rs.500. The friends who join also have to pool 4 more each to join so that they get a bicycle for themselves for Rs 100.

At the first glance there appears nothing wrong in the whole affair. The purchaser got the bicycle for Rs.100 as promised and the seller got his full price. Yet there is an obvious fraud. It caused losses to great many people who could not sell the scheme to the others. Initially it was easy to get others involved into this scheme but later

Many people got stranded after paying initial amount of Rs.100 and unable to complete the purchase. These people paid for the benefit of the early purchasers. The number of people drawn into the scheme increases as a geometric progression and that is not realized by initial enthusiasts. The number of people involved at every round is 1, 4, 20, 100, 500, 2500, 12500, 62500,

In the 12^{th} round the scheme will have involved the population of nearly the whole country of population 4 corers, four fifth of them loosing their money.

V. Lucky numbers

In a certain town car license plate had seven digits on them and digit 8 was considered unlucky. Accordingly a license number containing 8 as a digit was also considered unlucky. The seven digits on the plate can be any one among 0, 1, 2, 3, 4…. 9 and only one of them is unlucky so it was believed that unlucky numbers must be very few among the numbers on license plates. Indeed this is the case if the number had 6 or less digits. People watching the cars pass by would propose a bet ,“ I will pay you one rupee if it is lucky and you pay me one rupee if it is unlucky”. Those who did not bother to do the calculation fell to the temptation. The lucky numbers are less than unlucky ones in case of 7 digit numbers. Lucky ones are 8^7 – 1 = 2097151 in number and total possible numbers are 9^7 – 1 = 4782968. In both cases 0 is excluded as it can not be a number on license plate. This means lucky numbers are 2097151 and unlucky are 2685815. Chances of winning are little more than 56%.

VI. Dice gambling

Dicing was enormously popular in ancient Greece and Rome among the upper middle class. During the middle ages as Martin Gardner puts it, it was favorite time-waster for both the knights and the clergy. Later it became part of American casinos.

The player rolls two dice and wins if the sum is 7 or 11. He looses if the sum is 2, 3 or 12.

In other cases he gets a “point”. He then continues to throw. He wins if he throws a point before 7 and looses otherwise. If you actually do the calculation you will find that the chance of winning is approximately 0.493 and not ½.

VII. Candle Problem

Three friends A, B, and C are visiting a resort when the power gets cut off. A has 5 candles which they use. Then B takes out his stock of candles. They use 3 of them and the power comes back. C has no candles so he gives Rs.8 as his share. How do A and B share the money? Keeping the ratio 5:3 B gives Rs.5 to A and takes Rs.3 for himself. Is it fair? NO. It would be fair if only C had used the candle light. But A and B both also used the candle light for their benefit. When Rs.8 was estimated as value of everybody’s share, the total value was estimated to be 24 which means each candle costs Rs.3. A has contributed Rs.15 for the light which is 7 rupees more than his share and B has contributed Rs.9 which is Rs.1 more than his share. Hence A should get Rs.9 and B should get Rs.1.

VIII. Benjamin Franklin’s Will

In the year 1790 The American scientist Benjamin Franklin left £2000 in his will for the two cities he liked, Boston and Philadelphia. One year before he died he made an addition to his will and granted £1000 (which was approximately $4500 at that time) each to the cities close to his heart. The condition was that they must wait for 100 years to withdraw the money in part (75%) and wait for another 100 years to withdraw the full amount. Till then the money was to be used to give loans to young craftsmen to set up their business at the rate of 5%. Being a scientist Franklin knew that the money will grow substantially. Both the cities started following Franklin’s plan. After having used 75% of the funds after first 100 years at the end of 200 years i.e. in 1990 the city of Boston had 50 lakh dollars and city of Philadelphia had 20 lakh dollars. Boston did better because they diversified the investments. In 1990 the two cities shared the wealth generated and added it to their general funds. Through his will Franklin illustrated the tremendous power of the geometric series.

Use of e in Financial Mathematics e the constant used in Mathematics is defined as limit of $(1 + 1/n)^n$ as n tends to infinity. It can also be calculated as

$$e = 1 + \frac{1}{1} + \frac{1}{1 \times 2} + \frac{1}{1 \times 2 \times 3} + \frac{1}{1 \times 2 \times 3 \times 4} + \cdots$$

Its value is approximately 2.71828. Bernoulli noticed the use of e in financial Mathematics.
We know that the worth of an amount of Rs P at the rate r% after period of t year is $P(1 + r)^t$. But if the compounding is done n times in the year then the amount becomes $P(1 + r/n)^{nt}$. If n is very large then this approaches Pe^{rt}. This is the hypothetical case of continuous compounding. This formula is commonly used in financial Mathematics.

IX. Conclusion

Understanding of Mathematics is useful in daily life, not only the elementary arithmetic needed at the cash counter. It can guard us against frauds. People who dislike computing or do not know how to calculate are usually the ones who suffer in frauds. It is also useful in understanding and managing our finances.

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