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Multiplicity distributions for $p-n$ and $\bar{p}-n$ interactions at high energies.

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Abstract* - *Charged particles multiplicity distributions arising from $p-n$ and $\bar{p}-n$ collisions have been studied over the range of laboratory momenta from 50 to 400 GeV/c. The parton two fireball model (PTFM) based on an overlapping function and impact parameter analysis is adopted. The parameters controlling multiparticle production have been investigated and analyzed in the frame work of PTFM. Figures and calculations are provided to demonstrate good agreement between the theoretical calculations and the corresponding experimental data at different momenta.

***Keywords** – Charged pion production, Hadronic collision, Overlapping function, PTFM.*

I. Introduction

Many models have been used to identify and specify the hadron structure [1-3] along with other suggestions and assumptions to describe hadron-hadron interactions. These include fragmentation model [4, 5], the three fireball model [6], quark models [7], and many others. The theories and ideas concerning multiparticle production (especially pion production) go back to the late of 1930s with a significant interlude at Fermi's statistical theory of particle production [8]. Multiparticle production can be also modeled and described efficiently by studying the multiplicity distribution [9]. Several methods exist which investigate the multiplicity distribution of particles at high energy [10, 13]. Among these are the multiplicity scaling [10 11], the statistical boot strap model [12], the two sources model [14], the negative binomial distribution [15], fireballs [16], strings [17], quark gluon plasma [18, 19], and many others.

PTFM has been used in studying many cases; proton-proton, proton-nucleus and nucleus-nucleus interactions [20, 21]. All these studies showed good predictions of the measured parameters [22- 25]. In this work, we extend our model to study two another cases belonging to hadron-hadron interactions at high energies. The two cases are $p-n$ and $\bar{p}-n$ interactions at different momenta. Section 2 presents the proton neutron and antiproton neutron interactions at high energies. Section 3 presents the multiparticle production in proton neutron and antiproton neutron collisions. Section 4 presents c_q -moments of the charged particles multiplicity distributions. Section 5 provides the average charged particles multiplicity. Section 6 presents the dispersion of the charged particles multiplicity distributions. Section 7 provides the so-called KNO-Scaling. Section 8 presents the results and conclusion.

II. $p-n$ And $\bar{p}-n$ Interactions At High Energies

According to the parton two fireball model [20, 21, 26], $p-n$ and $\bar{p}-n$ interaction will be characterized by the impact parameter and the corresponding overlapping volume. Let us assume that the two interacting Hadrons at rest are spheres each of radius (R). Therefore the two colliding particles can interact strongly when the impact parameter is in the region from $0 \rightarrow 2R$.

Therefore, the statistical probability of any impact parameter (b) within an interval (db) is given by,

$$P(b)db = \frac{bdb}{2R^2} \qquad (1)$$

Let us use a dimensionless impact parameter, X, defined as, $X = \frac{b}{2R}$

Then, Eq. (1) can be rewritten as,

$$P(X)\,dX = 2X\,dX \qquad (2)$$

Where, $0 \le X \le 1$

Now we employ the overlapping volume, $V(b)$ as a clean cut [27] as,

$$V(b) = V_0\left(1 - \frac{3b}{8R} - \frac{3b^2}{8R^2} + \frac{5b^3}{32R^3}\right) \qquad (3)$$

In terms of the dimensionless impact parameter (X), the overlapping volume $V(X)$ can be given by,

$$V(X) = V_0\,(1 - 0.75X - 1.5X^2 + 1.25\,X^3). \qquad (4)$$

Then the fraction of partons, $Z(x)$ participating in the interaction may be written as,

$$Z(X) = \frac{V(X)}{V_0} = (1 - 0.75X - 1.5X^2 + 1.25X^3). \qquad (5)$$

According to Eq. (2) and Eq. (5), the Z-function distribution can be given by,

$$P(Z)dZ = 2XdZ\,(-2.4375 - 0.75\,X^{-1} + 7.125\,X + 0.75\,X^2 - 9.375\,X^3 + 4.6875\,X^4)^{-1} \qquad (6)$$

Where, $0 \le Z \le 1$

From Eqs. (2, 5) and using least square fitting technique (LSFT), Z-function distribution can be written in the following form,

$$P(Z)dZ = \sum_{k=-1}^{k=3} C_k Z^k dZ \qquad (7)$$

Where, C_k (k = -1, 0, 1, 2, 3) are free parameters to be calculated to produce a fitting between Eq. (6) and the curve drown from Eq. (7). From such fitting procedure the obtained values for C_k are,

$$C_{-1} = 0.089,\ C_0 = 1.21,\ C_1 = -2.65,\ C_2 = 3.228 \text{ and } C_3 = -1.823.$$

III. PION PRODUCTION IN $p-n$ AND $\overline{p}-n$ COLLISIONS

After the collision takes place, the partons within the overlapping volume stop in the center of mass system (CMS); their kinetic energy (K.E) will be changed into an excitation energy to produce two intermediate states (fireballs). The produced fireballs will radiate the excitation energy into a number of newly created particles, which are mostly pions. We assume that each fireball will decay in its own rest frame into a number of pions with an isotropic angular distribution. The number of created pions will be defined by the fireball rest mass (Mf) and the mean energy consumed in the creation of each pion (ε).

The energy available for the creation of pions from each fireball will be,

$$M_f - m = T_o\ \ Z(X) \qquad (8)$$

Where,

T_0 : is the kinetic energy of the incident proton in CMS and given by, $T_0 = \frac{Q}{2}$.

Q : is the total available kinetic energy in CMS.

The number of created pions (n_0) from each fireball will be given by,

$$n_0(\mathrm{Z}) = \frac{Z(X)T_0}{\varepsilon} = \frac{Z(X)Q}{2\varepsilon} \qquad (9)$$

It is clear that Eq. (9) gives the total number of created particles (charged and neutral) as a function of the dimensionless impact parameter.

To get the charged particles multiplicity distribution, we have to assume some distribution for the charged particles (n_{ch}) in the final state of the interaction at any impact parameter out from the total created particles (n_0). We considered the new created particles from each fireball can be divided into a number of pairs. Each pair will be either charged or neutral to satisfy the charge conservation.

From equations (7) and (9), the total number of created particles distribution, $P(n_0)$ can be calculated from the following equation,

$$P(n_o) = \sum_{k=0}^{3} (\frac{2\varepsilon}{Q})^{k+1} c_k \frac{(n_o+1)^{k+1} - n_o{}^{k+1}}{k+1} + c_{-1} \ln(\frac{n_o+1}{n_o}) \qquad (10)$$

If we assume a binomial and Poisson distributions for the probability distribution for the creation of charged pion pairs from one fireball of the forms,

1) Binomial distribution of the form,

$$\psi(n_2) = \frac{N!}{n_2!(N-n_2)!} p^{n_2} q^{(N-n_2)} \qquad (11)$$

2) Poisson distribution of the form,

$$\psi(n_2) = \frac{N^{n_2}}{n_2!} p^{n_2} e^{-NP} \qquad (12)$$

Where,

N : is the number of pairs of created particles from one fireball ($N = \frac{n_0}{2}$).

n_2 : the number of pairs of charged pions.
p : the probability that the pair of pions is charged
q : the probability that the pair of pions is neutral.
Therefore, the charged particles distribution from one fireball will be given by,

$$\varphi(\mathrm{n}) = \sum_{n_o} \psi(n_2)\, \mathrm{P}\ (n_o) \qquad (13)$$

Then, the charged particles multiplicity distribution from the two fireballs will be,

$$\mathrm{P}\,(\mathrm{n_{ch}}) = \sum_{n=1}^{nch} \varphi(n)\varphi(n_{ch}-n)\ ; \quad n_{ch} = 2,4,6,........,Q/\varepsilon \qquad (14)$$

If we assume that ε increases with the multiplicity size (n_0) as, $\varepsilon = a\, n_o + b$

Where, a and b are free parameters which can be taken to be, a = 0.02, b= 0.27.

Charged particles multiplicity distributions have been calculated at $P_L = 50{,}80$ GeV/c for $\overline{p}-n$ and $P_L = 100{,}200{,}400$ GeV/c for $p-n$ which are represented in fig. 1 a), b), c), d) and e) along with the corresponding experimental data [28-31].

IV. C_q -MOMENTS OF THE CHARGED MUL TIPLICITY DISTRIBUTIONS

The normalized moments c_q are defined by the relation,

$$c_q = < n_{ch}{}^q > / < n_{ch} >^q \qquad (15)$$

where q = 2, 3,………….., 10

Using the above relation, C_q -moments are calculated and represented in fig. 2 along with the available experimental data [32-38] which shows good fitting with the experimental data.

V. THE AVERAGE CHARGED PARTICLES MULTIPLICITY

From the charged particles multiplicity distributions, it is possible to calculate the average charged particles multiplicity by using the relation,

$$< n_{ch} > = \sum_{n_{ch}=2}^{Q|\varepsilon} n_{ch} P(n_{ch}) \qquad (16)$$

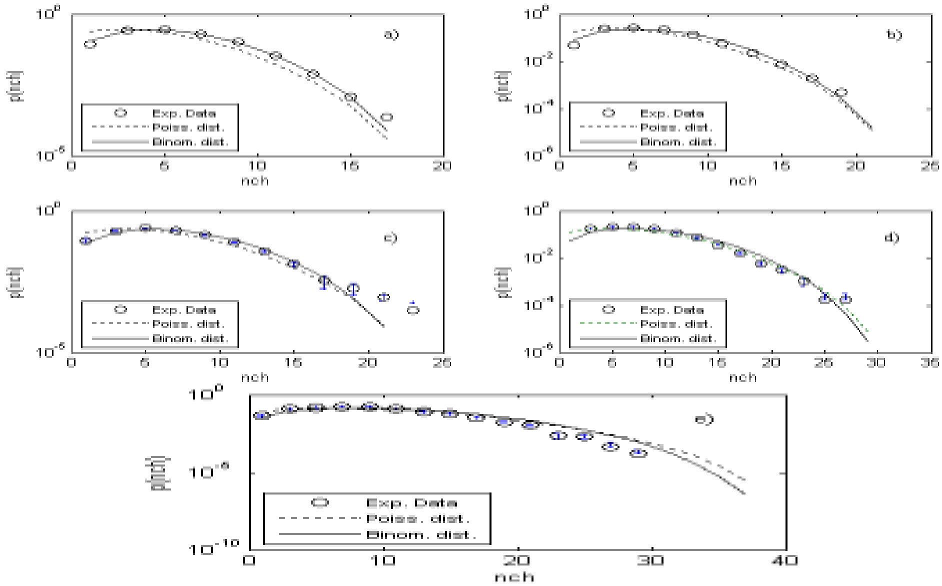

Fig.1. Multiplicity distribution of charged particles n_{ch} for $p-n$ and $\bar{p}-n$ collisions calculated

According to the parton two fireball model as parameterized by Poisson and binomial distribution in comparison with the corresponding experimental data at a) 50, b) 80, c) 100, d) 200 and e) 400 GeV/c

Thus, the multiplicity distributions of charged particles described above are used to calculate the average charged particles multiplicity at different incident momenta. These calculations are represented in fig. 3 along with the available experimental data [28-31] which shows good agreement with the experimental values.

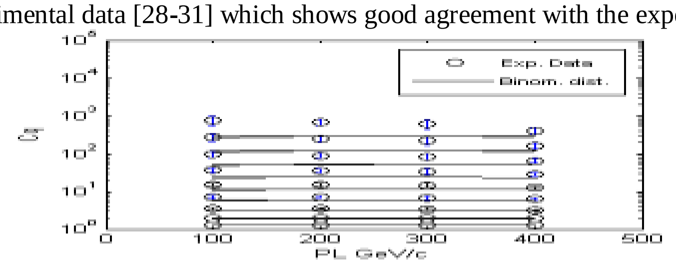

Fig.2. The dependence of C_q moments for $p-n$ collisions at 100-400 GeV/c as a function of the incident momenta

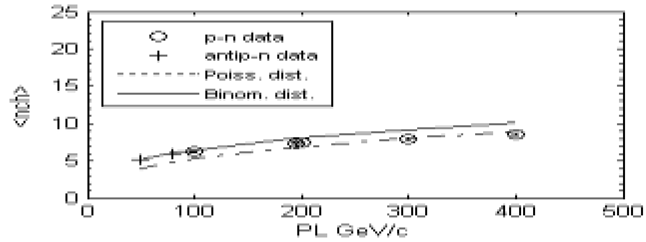

Fig.3. Variation of average number of charged particles $< n_{ch} >$ for $p-n$ and $\bar{p}-n$ versus the incident momentum predicted by the parton two fireball model

VI. DISPERSION

The dispersion of the multiplicity distribution is defined as,

$$D = \left(< n_{ch}{}^{2} > - < n_{ch} >^{2}\right)^{1/2} \qquad (17)$$

and the dispersion parameter is $< n_{ch} > / D$ which is found experimentally to be approximately

constant with the increase in the incident momenta.

We have calculated the charged multiplicity moments, $< n_{ch}{}^{2} >$, $< n_{ch} >^{2}$ and hence the dispersion parameter $< n_{ch} > / D$, the results obtained at different momenta are given in fig. 4 together with the available

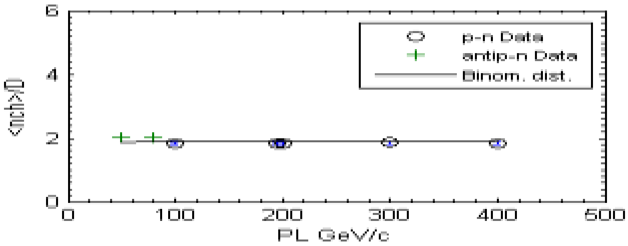

Fig. 4. Dispersion for $p-n$ and $\bar{p}-n$ versus P_L GeV/c

Data [28-31]. It can be seen from the figure that the predictions are in excellent agreement with observations.

We have also calculated the correlation between dispersion parameter and average multiplicity; the results obtained are given in fig.5 together with the available data [28-31]. It can be seen from this figure that the predictions are in excellent agreement with observations.

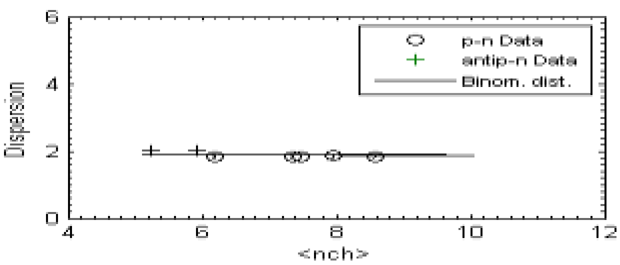

Fig.5. Dispersion versus $< n_{ch} >$

KNO-SCALING

An interesting feature of the topological cross section is the idea of KNO- scaling [11] suggested by Koba,

Nielson and Olesson. According to KNO - Scaling, if we plot the relation between multiplicity distributions of charged particles based on the above scheme multiplied by the average charged particles multiplicity $< n_{ch} > P(n_{ch})$ and the number of charged particles divided by the same quantity $n_{ch} / < n_{ch} >$ at different momenta, then, the relation must be energy independent.. These calculations are represented in fig. 6.The obtained results show good agreement with the corresponding experimental data [27-29].

VII. CONCLUSION

The charged particles multiplicity distributions, Eq. (14), are calculated for $p-n$ and $\bar{p}-n$ assuming ε is given by, $\varepsilon = a\, n_o + b$

Where, a = 0.02, b = 0.27. The results of these calculations are represented in fig. 1 a), b), c), d) and e) along with the experimental data [28-31] which show good agreement with the corresponding experimental data. It can be seen from fig.1 that the emission of secondary particles is assumed to follow a binomial distribution. Fig.2 shows the dependence of C_q moments for $p-n$ collisions at 100-400 GeV/c on incident momenta and shows also a good fitting between our calculations and the corresponding experimental data [32-38]. Fig.3 shows the variation of the average charged particles multiplicity $< n_{ch} >$ with various laboratory momenta (50, 80, 100, 200 and 400 GeV/c), from fig.3 it can be seen that the dependence of $< n_{ch} >$ on laboratory momenta is in accordance with the experimental data [28-31]. Figs.4, 5 show the dependence of dispersion on incident momenta and the correlation between dispersion and the average multiplicity and the experimental data [28-31] are consistent with our predictions. Fig.6 views the multiplicity distribution of charged particles n_{ch} for $p-n$ collisions at momenta ranges 100-400 GeV/c using the parton two fireball model in the form of KNO-Scaling in comparison with the corresponding experimental data [28-31] and that scaling is clearly energy independent.

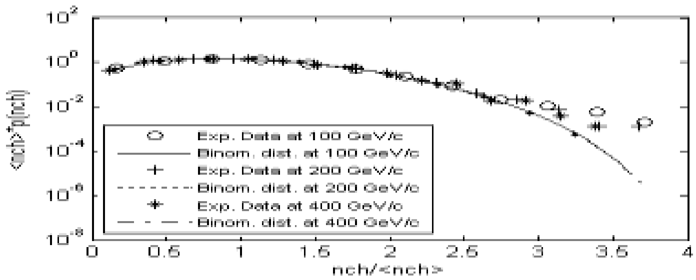

Fig.6. Multiplicity distribution of charged particles n_{ch} for $p-n$ collisions at energies ranges

100-400 GeV/c using the parton two fireball model in the form of KNO-Scaling in comparison with the corresponding experimental data

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