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Contribution to the Understanding of the Difference between Values for Theoretical and Experimental Transverse Modulus of Uniaxial Fibrous Composites

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Abstract
In this article is discussed the challenges in obtaining accurate experimental data for transverse modulus polymer fiber reinforced composites. The authors analyze data epoxy composites with glass and carbon fibers and propose ways to understand the difference between theoretical and experimental values. Theoretical models for calculating the transverse modulus should consider the difference between young's modulus of matrix and fiber, individual transverse modulus values, and matrix load transmission ability to the fiber, which decreases with increase of the fiber content.
Key words: *Polymer composites. Polymer fiber reinforced composites. Transverse modulus of fiber reinforced composites.*

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I. Introduction

Isotropic and homogeneous materials have equal properties along all directions in all planes of symmetry, exhibiting equal modulus under tension and compression, however the simplest of fibrous composites, a lamina composed of an isotropic matrix uniaxially reinforced by isotropic fibers (see Fig. 1), exhibits a high degree of anisotropy [1].

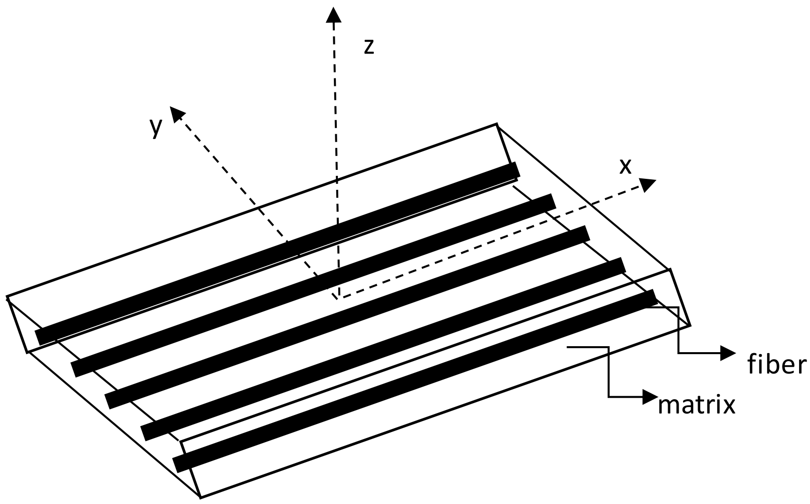


Figure 1. polymer matrix lamina with unidirectional fibrous reinforcement.

Fibrous reinforcement, in general, increases the modulus of the system. This phenomenon happens due to the high resistance that the fibers have along their length, and this increase will depend on the several factors [2-10]. If the fiber reinforcement is continuous, that is, if the fibers are regularly dispersed in the matrix and oriented along the length, the mechanical properties measured in this direction will be greater than in perpendicular directions or inclined by some angle in relation to the length.

Polymers are well-known materials in advanced applications for many years. They are versatile materials and easy to be molded into any required application. However, there are a few aspects in the field of polymer to consider well, a single polymer cannot meet the requirements in advance applications. Therefore, polymer composites attracted the attention of the world [11].

A composite consists of at least two parts, one is reinforcement and the second is the matrix. The composite may contain metals, ceramics, and other polymers as a matrix and as reinforcement. In polymer composite, thermosetting and thermoplastic resins have been used extensively as the matrix. The thermosets are of low viscosity, while thermoplastics have the possibility of recycling and reuse. Essentially all commercially important polymers have advance applications. Polymer composites are a rapidly growing industry are used mainly in automotive and aerospace applications [11-12].

In a Polymer composites, a polymeric resin penetrates the reinforcement bundles and bonds to the reinforcement, but this bond between matrix and reinforcement is not always effective and the diffusion of load from a fiber to a surrounding matrix depends on interface stiffness [12].

The structural polymer composites, basically consisting of continuous fibers, considering their high strength, high modulus, and low density, are dominant among lightweight structural composites. Today fiber-reinforced polymer composites (FRPs) and their large-scale manufacturing techniques are of particular interest. Two main groups of FRPs are glass fiber-reinforced polymer composites and carbon fiber-reinforced polymer composites [13].

The transverse modulus of composites reinforced by unidirectional fibers is a mechanical property that has generated significant controversy. This work aims to analyze the existing experimental data for the transverse modulus of epoxy composites reinforced with both glass and carbon fibers, and compare them with values obtained from Voigt's idealized model.

Theoretical Approaches

The elastic modulus of an isotropic and homogeneous material is related to the interatomic bonding strength as follows:

$$F = \frac{dU}{dr} \quad (1)$$

Where F is the force of a pair of atoms separated by a distance r. The rigidity S of the bond between the two atoms is obtained by

$$S = \frac{dF}{dr} = \frac{d^2U}{dr^2} \quad (2)$$

At the equilibrium distance, r_0, the stiffness is constant, that is,

$$S_0 = \left(\frac{d^2U}{dr^2}\right)_{r=r_0} \quad (3)$$

If force, F, is applied over an average atomic area, r_o^2, and promotes a displacement such that $(r - r_{0)}$ tends to zero, we have that

$$F = S_0(r - r_{0)} \quad (4)$$

If F is divided by r_o^2, we have a tensile stress given by

$$\sigma = \frac{S_0(r - r_{0)}}{r_o^2} \quad (5)$$

this tensile stress will produce tensile strain

$$\varepsilon = \frac{(r - r_{0)}}{r_0} \quad (6)$$

If we compare Eqs. (5) and (6) we get

$$\sigma = \left(\frac{S_0}{r_0}\right)\varepsilon \quad (7)$$

Hooke's law for elasticity dictates that

$$\sigma = E\varepsilon \quad (8)$$

the comparison between Eqs. (7) and (8) shows that Young's theoretical module is given by

$$E = \left(\frac{S_0}{r_0}\right) \quad (9)$$

The ideal composite imagined by Voigt is the one that has its properties, specially the modulus, with intermediate values to those of its constituents. Assuming compatible isotropic reinforcement and matrix and in the absence of voids or delamination, it can be demonstrated that (see ref. [2]):

$$\frac{1}{E_Y} = \frac{\emptyset_F}{E_F} + \frac{\emptyset_M}{E_M} \quad (10)$$

where E_Y is the transverse modulus of the composite, E_F is the isotropic modulus of the fiber and E_M is the isotropic modulus of the matrix. $\emptyset_F$ and $\emptyset_M$ the volumetric fractions of fibers and matrix, respectively. With

$$\emptyset_F + \emptyset_M = 1 \quad (11)$$

Real composites can hardly have the modulus obtained by Eq. (10), even if several of the initial assumptions are obeyed. Thus, several theoretical, semi-empirical and empirical models were proposed (part of the models will be shown in this work) with the aim of better correlating the parameters presented in Eq. (10).

Effective Concentration Approach

The first approach involves analyzing the respective contributions of the matrix and fiber in reinforcing the composite. This approach suggests that the effective concentration of the constituents may be greater than what is actually present in the composite. This is due to the fact that the constituent may transmit a property that is either superior or inferior to the estimated value for that volumetric fraction. Like this,

$$E_Y^{1-n} = \frac{E_{ef}}{E_L^n} \quad (12)$$

where the transverse modulus, E_Y, would be obtained from an effective modulus of the composite, E_{ef}, and the modulus of the composite with the longitudinal arrangement of the fibers, E_L, with n being a parameter obtained by fitting the curve E_Y x $\emptyset_F$, given by [3]:

$$n = \frac{\ln\left(\frac{\emptyset_M}{E_M}E_F + \frac{\emptyset_F}{E_F}E_{ef}\right)}{\ln\left[1 + \emptyset_M\emptyset_F\left(\frac{E_M}{E_F} + \frac{E_F}{E_M} - 2\right)\right]} \quad (13)$$

Iso-Stress Approach

This approach results in a model (ISM) that provides a lower bound estimative for transverse modulus by virtue of its assumption of homogeneous uniaxial transverse stress and nonhomogeneous multiaxial strains throughout the composite [4].

The ISM transverse modulus is given by

$$\frac{1}{E_Y} = \frac{\emptyset_M}{E_M^{ef}} + \frac{\emptyset_F}{E_F} \quad (14)$$

where E_M^{ef} is the Young's transverse modulus of a matrix without voids or imperfections.

Debonding Matrix-Fiber Effect

Shan & Chou [5] developed an elastic contact model to predict E_Y and other elastic constants of unidirectional fiber composites with interfacial debonding. They found that the transverse young's modulus of a debonded composite in compression is higher than that in tension, and lower than that of a perfect bonded composite and that this modulus is almost equal to that of matrix containing fiber-like voids.

In a separate study, Spencer [6] proposed that cracking, yield/creep, or debonding occurs immediately between fibers due to stress concentration where the resin thickness is minimal. He also noted that this phenomenon becomes more severe as the fiber modulus increases. For this case, he proposed

$$\frac{E_Y}{E_M} = \frac{\gamma - 1}{\gamma} + \frac{1}{k}\left[-\frac{\pi}{2} + \frac{2\gamma}{\sqrt{\gamma^2 - k^2}}\tan^{-1}\left(\sqrt{\frac{\gamma + k}{\gamma - k}}\right)\right] \quad (15)$$

where

$$\gamma = \left(\sqrt{(1.1\emptyset_F^2 + 2.1\emptyset_F + 2.2)\emptyset_F}\right)^{-1}$$

and

$$k = 1 - \frac{E_M}{E_F}$$

Voids Effect

In fibrous composites, particularly those molded without additional pressure, the presence of voids in both the matrix and the matrix-fiber interface is common. This leads to stress concentration [6]. In such cases, the volumetric fraction of the matrix must be reduced by the amount corresponding to the voids. Consequently, Eq. (10) is modified as follows [3]:

$$\frac{1}{E_Y} = \frac{\emptyset_F}{E_F} + \frac{(\emptyset_M + \emptyset_V)}{E_M^Y} \quad (16)$$

where, $\emptyset_V$ is the void volume fraction and E_M^Y is the matrix transverse modulus.
If the composite is hybrid and composed of two types (a and b) of fibers, for example, Eq. (16) becomes [3]:

$$\frac{1}{E_Y} = \frac{\emptyset_F^a}{E_F^a} + \frac{\emptyset_F^b}{E_F^b} + \frac{(\emptyset_M + \emptyset_V)}{E_M^Y} \quad (17)$$

Micromechanical Approach

Fu et al. [7] proposed a micromechanical model for predicting the transverse modulus of unidirectional continuous and discontinuous fiber composites. This model is based on modeling a composite with regular array of volume elements, constructing a stress pattern based on simple averaging procedures in the direction transverse to the fiber axis for a representative volume element. In this model, E_Y is given by

$$\frac{1}{E_Y} = \frac{\dfrac{E_M}{\sqrt{(4\emptyset_F/\pi)}}}{\sqrt{(\pi\emptyset_F/4)E_y^F + \left(1 - \sqrt{(\pi\emptyset_F/4)}\right)E_M}} + \frac{\left(1 - \sqrt{(\pi\emptyset_F/4)}\right)}{E_M} \quad (18)$$

Verkatean et al. [3] pointed out that some models lack fitting parameters, while others significantly underpredict experimental data and do not agree with actual values. Although some models show good agreement, they are difficult to use due to a large number of parameters that require fitting. Furthermore, none of the proposed models utilize properties of the fiber or matrix other than those related to mechanical or geometric viewpoints.

This paper compares experimental data on the transverse modulus of fiber composites, presented in various publications, with data provided by the ideal model proposed by Voigt, as shown in Eq. (10). The discussion centers around the matrix's ability to transfer load to fibers or the compatibility of the matrix-fiber pair within the composite.

Analysis of Experimental Data

To analyze the experimental data, we selected materials with sufficient data on the transverse modulus, and concentrations that can be compared with each other. Therefore, we chose Epoxy as the matrix reinforced by either glass or carbon fibers. The relevant data for these materials can be found in the indicated references.
We will call the theoretical transverse modulus, E_y^T, the one calculated by Eq. (10). The values present in published works will be called E_y^X. For initial comparison purposes, the quantity α will be introduced, which will be the absolute percentage error between the theoretical and experimental values given by:

$$\alpha = \left|\frac{E_y^X - E_y^T}{E_y^T}\right| \quad (19)$$

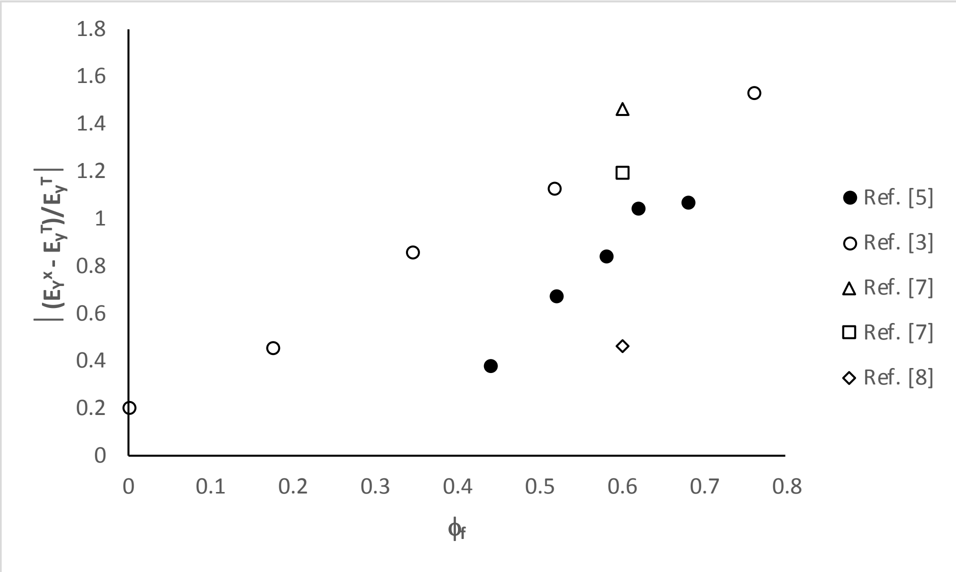


Figure 2. Difference between theoretical and experimental transverse moduli for glass fiber reinforced epoxy composites as a function of fiber content.

As can be seen in Figure 2, the difference between the experimental values and those obtained by Eq. (10) increases with glass fiber content. When comparing hybrid composites with glass fiber and carbon fiber, see Ref. [3], it is observed that this difference decreases with increasing carbon fiber content, as shown in Figure 3. This suggests that there is a certain compatibility between epoxy and carbon fiber greater than that existing between epoxy and glass fiber, since the preparation method was the same for the composite with both fibers.

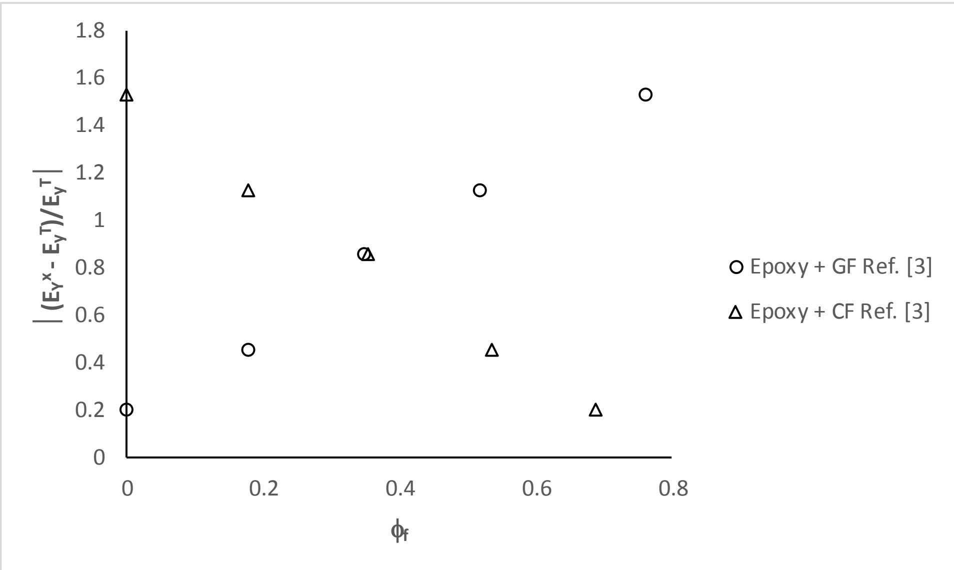


Figure 3. Difference between transverse modules for carbon fiber and glass fiber reinforced epoxy composites as a function of fiber content.

Reinforcement	E (GPa)	E_y^T (GPa)	E_y^X (GPa)
E-glass	72.3	8.2	12
Kevlar 49	124	8.4	5.5
Carbon T300	218	8.5	10.3
Carbon GY49	531	8.7	5.5

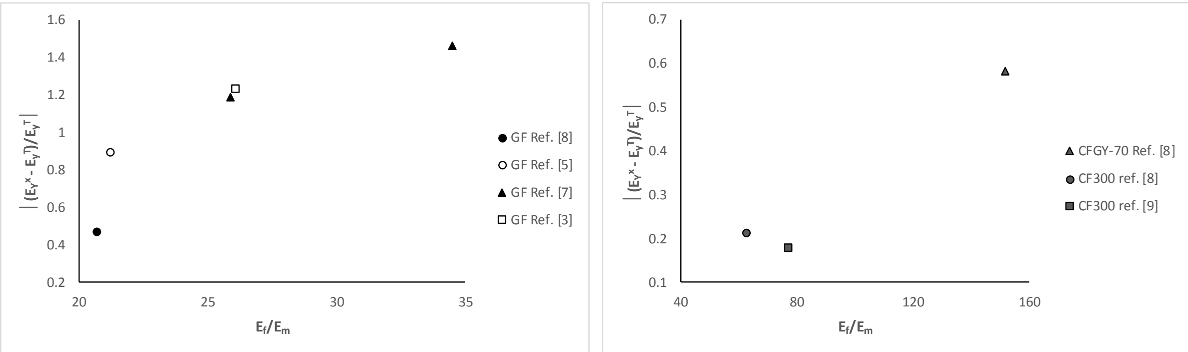
Table 1. Experimental and theoretical values of transverse modulus of epoxy composites (E_M= 3.45 GPa) with various fibers with $\emptyset_F$ of 0.6. [8].

In Dowling's book [8], it is observed that different values of α are obtained for the same fiber concentration in an epoxy matrix composite, especially for carbon fibers with different isotropic moduli. Table 1 presents these data.

From the data in Table 1, especially in the case of the epoxy-carbon fiber composite, it is possible to infer that there may be a relationship between the difference between the theoretical and experimental values of E_Y with the ratio between fiber, E_F, and matrix, E_M, Young's modulus, i.e.,

$$\alpha \sim \frac{E_F}{E_M} \quad (20)$$

Figures 4a and 4b do not corroborate the influence of the possible compatibility between matrix and fiber, since, both for the epoxy-glass fiber and epoxy-carbon fiber composites, alpha increases with E_F/E_M. In this case, the difference between the theoretical and experimental values for the transverse modulus is directly related to the ability of the matrix to deliver the external load to the fiber rather than any chemical affinity that may exist between them. This capacity, according to these observations, will be greater the smaller the E_F/E_M ratio.

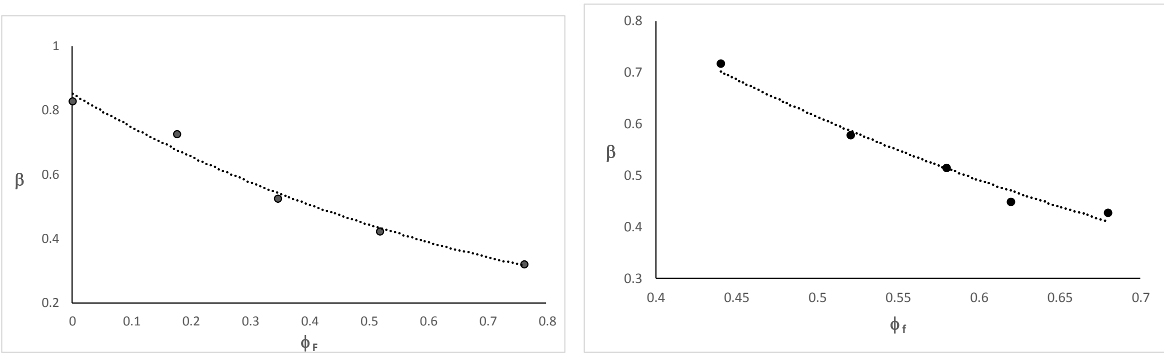


(a) (b)

Figure 4. α variation with E_F/E_M for epoxy and glass fiber composite (a) and epoxy and carbon fiber composite (b).

If it is introduced in Eq. (10) a β factor, so that

$$\frac{1}{E_y^x} = \frac{\emptyset_F}{E_F} + \beta \frac{\emptyset_M}{E_M} \quad (21)$$



(a) (b)

Figure 5. Variation of the β-factor of the epoxy-glass fiber composite with the glass fiber content according to (a) Ref. [3] ($E_M = 3.07\ GPa;\ E_F = 80\ GPa$) and (b) Ref. [5] ($E_M = 3.45\ GPa;\ E_F = 73.1\ GPa$).

and applying it to the epoxy-glass fiber systems of Refs. [3] and [7], we obtain the data present in Figures 5a and 5b.

Both cases shown in Figure 5 reveal a decrease in the β factor with increasing fiber content in the composite, indicating that a higher fiber content poses greater difficulty for the matrix to transfer external loads to the fibers. This outcome is expected since a higher $\emptyset_F$ value reduces the volumetric concentration of the matrix in the composite, allowing the properties of the fiber to dominate over those of the matrix. Previous studies [8,10] have shown that the transverse modulus of the fiber is significantly lower than its longitudinal modulus, which is anticipated due to the molecular orientation of the fibers in their length direction. Since nearly all proposed models for calculating the transverse modulus of fibrous composites use the longitudinal modulus values of the constituents, they would inevitably yield results that are incompatible with experimentally observed values.

The statistical fit for $\beta \; x \; \emptyset_F$, shown by the dotted line in Figure 5, indicates that the dependence $\beta(\emptyset_F)$ obeys an exponential function. If we plot $\beta.\emptyset_M$ versus $\emptyset_F$ for the system from Figure b, for example, we get:

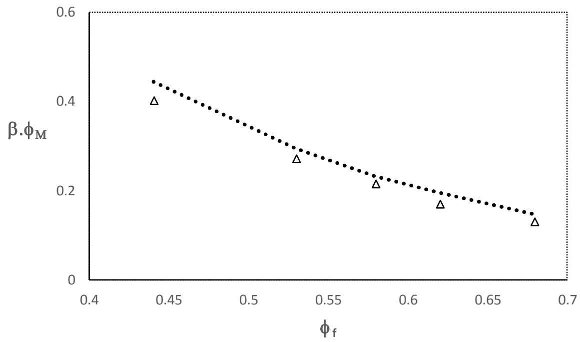


Figure 6. $\beta.\emptyset_M$ versus $\emptyset_F$ for system of Figure 5b. Where Δ are data obtained from [3] and by use of Eq. (21), and ▪▪▪▪▪▪ for data obtained from Eq. (22).

The statistical fit of data from Figure 6 shows that:

$$\beta.\emptyset_M = \left[\sqrt{\left(\frac{E_F \upsilon_F}{E_M \upsilon_M}\right)} - \frac{\upsilon_M + \upsilon_F}{2}\right] e^{-\sqrt{\frac{E_F}{E_M}}.\emptyset_F} \quad (22)$$

where, υ_M e υ_F are Poisson coefficients of matrix and fiber respectively.

From Eq. (22):
$a)$ $\beta = 1.0 \; if \; \emptyset_F = 0$ and $E_Y = E_M$
b) $\lim_{\emptyset_M \to 0} \beta.\emptyset_M = 0$ and $E_Y = E_F$
Introducing Eq. (22) in Eq. (21), we have

$$\frac{1}{E_Y} = \frac{\emptyset_F}{E_F} + \frac{1}{E_M}\left[\sqrt{\left(\frac{E_F \upsilon_F}{E_M \upsilon_M}\right)} - \frac{\upsilon_M + \upsilon_F}{2}\right] e^{-\sqrt{\frac{E_F}{E_M}}.\emptyset_F} \quad (23)$$

As can be seen in Figure 7, the model based on the β factor is adequate to adjust the experimental data for 0.4 < $\emptyset_F$ < 0.7, while the Voigt model is only for values of $\emptyset_F$ tending to 0.0. Since most polymer matrix composites use about 60% fiber as reinforcement, the model base on the β factor can be a suitable tool for predicting the transverse modulus of composites whose behavior of the matrix-fiber pair is previously known.

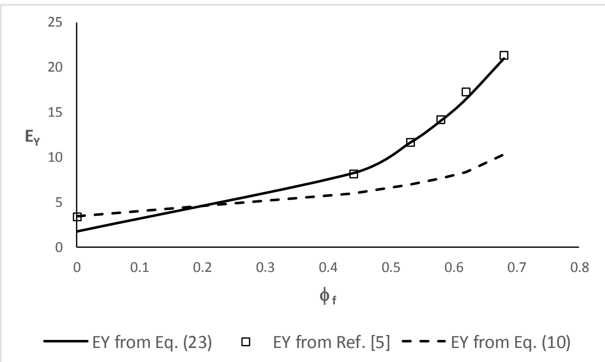


Figure 7. Experimental data fitted by Eq. (14) and by Eq. (21) combined with Eq. (23) for the system from the Figure 5b.

II. Conclusion

In conclusion, this study focused on analyzing the transverse modulus of epoxy composites with glass and carbon fibers. The results showed a significant difference between theoretical and experimental values, which can be attributed to several factors including the difference in Young's modulus between the matrix and the fiber, as well as the ability of the matrix to transfer load to the fiber. The study highlights the need for theoretical models to consider these factors in order to accurately predict the transverse modulus of fibrous composites. Overall, this work contributes to a better understanding of the behavior of polymeric matrix fibrous composites and provides insights for future studies in this field.

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