*Research Inventy: International Journal of Engineering And Science*
*Vol.11, Issue 7 (July 2021), PP 22-24*
*Issn (e): 2278-4721, Issn (p):2319-6483, www.researchinventy.com*

# Controlling Cart and Pole to Move Through Rotating Pinwheel

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***Abstract:*** *Balancing inverted pendulum on cart is classical problem that is solved in many researches. However, there is no study about the operation of this model in contacting with environment to satisfy a requirement. Thence, in this paper, we suggest a problem of controlling cart and pole which needs to move through an operating pinwheel without touching the wings of pinwheel. We propse the parameters for total structure. Then, a process of calculating the requirement of velocity of rotating pinwheel is described. The process of solving this mathematical problem is a reference for similar researches.*
***Keywords:*** *Cart and pole, inverted pendulum, pinwheel, balancing control.*

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Date of Submission: 28-06-2021 Date of Acceptance: 12-07-2021

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## I. INTRODUCTION

Cart and pole (C&P) is a popular model in control automation [1]. Its other name is inverted pendulum on cart. However, many researches about this model are just focused on the balancing of the pole [2], [3]. There is no study about the operation of whole C&P in environment. In other to create the more general situation in which C&P has to work well with requirement of environment while balancing cart, we widen the problem as in Figure 1:

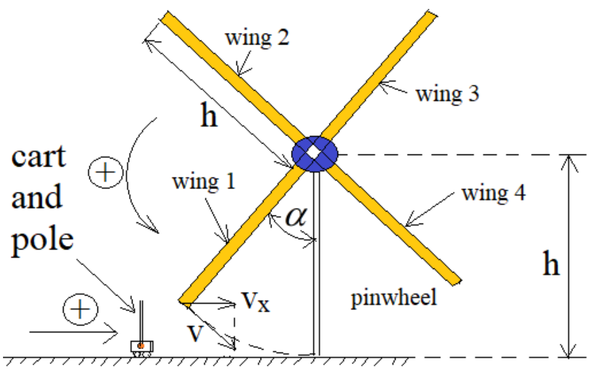

**Figure 1.** Assumption of platform of problem

- There is a pinwheel which is locate on a side of the path. The C&P can go on the path without touching the pole which connects piwheel with the ground.
- The lenghth (h) of wing is approaximatly the height of pole of pinwheel. The wing nearly touches the ground when it is in lowest position (Figure 3).
- The pinwheel has four wings. These wings rotate with a stable constant angle velocity as

$$\omega = const \tag{1}$$

Then, speed of top of wing is

$$v = \omega h \tag{2}$$

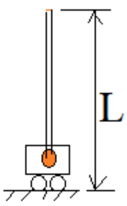

**Figure 2**. Size of C&P

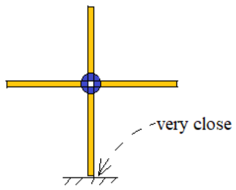

**Figure 3.** Position of wings when being very closed to the ground

- The C&P has to move faster to go through the wings while they working before the pole is hit by the wings.
- Purpose is that cart and pole moves through position of pinwheel without touching the wings that rotates.
- The size of cart is unremarkable. The lenght of pole (L) is remarkable (Figure 3).

The problem we need to solve is:
+ In what situation, the C&P have to contact the operating space of pinwheel?
+ $\omega = ?$

## II. CALCULATIONS

From Figure 1, horizontal speed of top of wing is

$$v_x = v\cos\alpha = \omega h \cos\alpha \tag{3}$$

The operating space of pinwheel is chosen to be the period when the wing 1 has gone over and the wing 2 just begins to move behind the C&P as in Figure 4. This period ends when system is as in Figure 5.

From Figure 4, we obtain:

$$h = h\cos\alpha_1 + L \rightarrow \alpha_1 = ar\cos\left(1-\frac{L}{h}\right) \quad (4)$$

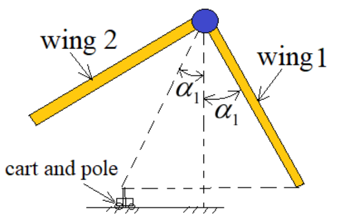


**Figure 4.** Condition of C&P beginning going into the space that it is affected by pinwheel

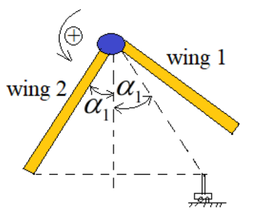


**Figure 5.** Condition of C&P beginning going out the space that it is affected by pinwheel

The distance that the cart need to finish from Figure 4 to Figure 5 is

$$S = 2h\sin\alpha_1 \quad (5)$$

Time that cart need to send when moving through that distant is

$$t = \frac{S}{v_{set}} = \frac{2h\sin\alpha_1}{v_{set}} \quad (6)$$

where $v_{set}$ is the velocity that we want the cart to follow.

From geometrical alculations from Figure 5, the parameter t in (6) can be obtained as

$$t = \frac{\frac{\pi}{2} - 2\alpha_1}{\omega} \quad (7)$$

From (6), (7), after calculations, we obtain

$$v_{set} = \frac{2\omega h\sin\alpha_1}{\frac{\pi}{2} - 2\alpha_1} \quad (8)$$

System paramaters of C&P and motor are listed in Table 1 and Table 2 [4]

**Table 1.** System parameters of C&P

| Parameters | Descriptions | Values |
|---|---|---|
| M | Mass of Cart | 0.1 (kg) |
| m | Mass of inverted pendulum | 0.01 (kg) |
| L | Lenght of pendulum | 0.5(m) |
| g | Gravitational acceleration | 9.81($m/s^2$) |
| $C_1$ | Distance between center of pendulum and the rotating axis of pendulum | $\approx \frac{L}{2} = 0.25$ (m) |
| $J_1$ | Inertial moment of inverted pendulum | $\approx 0$ |

**Table 2.** Parameters of DC motor that control C&P

| Para-meters | Descriptions | Values |
|---|---|---|
| $R_m$ | Resistor of motor | 8.4(Ω) |
| $L_m$ | Reactance | $1.16\text{x}10^{-3}$(H) |
| $K_b$ | Backclash constant | 0.042 (Vsec/rad) |
| $K_t$ | Inertial constant | 0.042 (Nm/A) |
| $J_m$ | Inertial moment of motor | $4\text{x}10^{-6}$(kgm) |
| $C_m$ | coefficient of viscous friction | $\approx 0$ (Nmsec/rad) |
| $T_f$ | Friction constant | $\approx 0$ (Nm) |

Also from [4], dynamics equations of C&P are

$$M_f(q)\ddot{q} + V_{mf}(q,\dot{q})\dot{q} + G_f(q) = [k_1 e \quad 0]^T \quad (9)$$

where:

$$q = \begin{bmatrix} x \\ \theta \end{bmatrix},\ M_f(q) = \begin{bmatrix} m+M+k_3 & mC_1\cos\theta \\ mC_1\cos\theta & J_1 + mC_1^2 \end{bmatrix},\ V_{mf}(q) = \begin{bmatrix} k_2 & -mC_1\dot{\theta}^2\sin\theta \\ 0 & 0 \end{bmatrix},\ G_f(q) = \begin{bmatrix} 0 \\ -mC_1 g\sin\theta \end{bmatrix}$$

This C&P can be controlled by PID controller. By using genetic algorithm, the control paramaters are:

$$K_{p1} = 94.5,\ K_{i1} = 16.2,\ K_{d1} = 39.8,\ K_{p2} = 96.5,\ K_{i2} = 51.7,\ K_{d2} = 13.7 \quad (10)$$

By choosing set point 0.1m, cart need 20s to be at expected position (through Matlab/Simulink simulation). Then, it yields

$$v_{set} = \frac{0.1}{20} = 0.005(m/s) \quad (11)$$

From (8) and (11), after calculations, we obtain:

$$(v_{set} =)\ \frac{2\omega h \sin \alpha_1}{\frac{\pi}{2} - 2\alpha_1} = 0.05 \qquad (12)$$

Choosing $\alpha_1 = 20^0$ , from (4), we have h=8.2909(m)

Thence, from (12), velocity of wing of pinwheel should be chosen as $\omega$=9x10$^{-5}$(rad/s)=0.052$^0$/s.

## III. CONCLUSION

In this paper, we widen the problem of balancing system to wider situation. The C&P not only work well in balancing the pole but also move through an operating space. In this space, the wings of pinweheel rotates without touching the pendulum (pole). In order to satisfy those requirements, we prose the system parameters of whole platform (C&P, DC motor, pinwheel), PID controller, set points and conditions of experiment. By calculations, genetic algorithm and Matlab/Simulink simulation, we prove the possibility of this platform.

**Conflict of interest**

There is no conflict to disclose.

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Van-Dong-Hai Nguyen. "Controlling Cart and Pole to Move Through Rotating Pinwheel." *International Journal of Engineering and Science*, vol. 11, no. 7, 2021, pp. 22-24.