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# Effectiveness of Thickness Variation on Deformations for Plate Bending Problems

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***Abstract:*** *The intention of this study is to extend analytical solutions of the discrete one-dimensional beam elements for solution of plate bending problems with respect to thickness variations. The solution can be stated as an extension of the so-called discrete parameter approach where the plate elements broken down into discrete sub-domains. The differences in the structural performance levels determined for the structural elements. The finite grid method was developed to determine deformation of rectangular thin plates subjected to external load, perpendicular direction to the plane, provides a great advantage considering the flexible boundary conditions and external loading. Elastic thin plates are used in many engineering structures. In this study, the maximum deflection values of thin rectangular plates were calculated by finite grid method and other solution methods were compared with the analytical solutions as case study. The analyzes are determined by Sap2000 and Ansys Workbench with difficulties in case of complicate plate problems for mathematical modeling. It is noted that the method is a powerful preferable method for complex plate bending problems with thickness variations that provides small error rates.*

***Keywords:*** *Plate Bending; Thickness Variation; Finite Grid Method; Deformations in Bending.*

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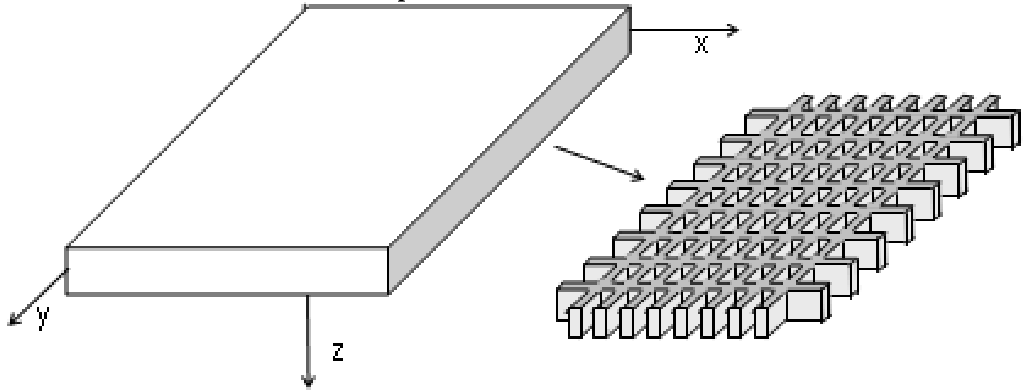
## I. INTRODUCTION

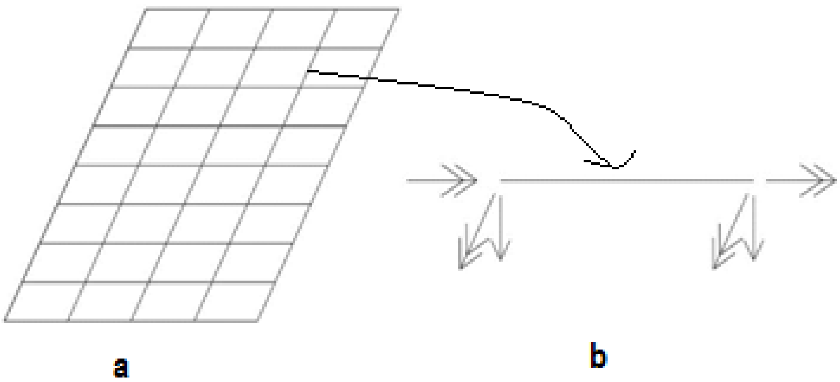
The two dimensional plates can be modeled as a particular case of three-dimensional continuum by eliminating the thickness direction. The two-dimensional plate structures can be analyzed Such integration can be made by following two different methods well known examples of classical thin plate theories that have been derived on the basis of intuitions are the 'Thin Plate Theory' (TPT) by Cauchy ( Cauchy, 1828) and Kirchhoff (Kirchhoff ,1850) type assumptions; discard transverse shear and trough-the-thickness deformation. After all, within limitations of simplified formulation as Wilson [indicated, plate bending is an extension of one dimensional beam theory(Wilson, 2002). In present study the rectangular plates through the lattice analogy at which the discrete elements are connected at finite nodal points is represented by one dimensional beam elements.The individual element matrices are used to form the system load and stiffness matrices for plates (Karasin et al., 2017. The matrix displacement method based on stiffness-matrix approach is suitable to solve gridworks with arbitrary load and boundary conditions. A combination of finite element method, lattice analogy and matrix displacement analysis of grid works was used to obtain a finite grid solution (Akdogan and Karasin, 2016; Karasin et al., 2018). This study is oriented toward the development of finite grid element. It is an application of the finite element method. The aim is to investigate an improved finite grid solution for vibration problems of plates on elastic foundation. This is possible for free as well as forced vibration cases. The solution can be stated as an extension of the so-called discrete parameter approach where the physical domain is broken down into discrete sub-domains, each endowed with a response suitable for the purpose of mimicking problem at hand. In another words this method the discretized plate element is reassembled by the matrix displacement method so that consistent mass matrix of the total structure is generated schme to compute all displacements for each nodal point in a convenient sequence. By this representation, it is possible to solve complicated plate problems such as non-uniform thickness arbitrary boundary and loading conditions and discontinuous surfaces.The system cannot truly be equal to the continuous structure but solutions adequate for engineering purposes can be found with greater ease. In this form, plates are idealized as a grillage of beams of a given geometry satisfying given boundary conditions as shown in Figure 1.

**Figure 1. Representation of a typical rectangular plate by grids as parallel sets of one-dimensional beam elements replace the continuous surface.**




It is noted that the discreated elements which **replace the continuous surface** are connected at finite nodal points of a rectangular thin plate in flexure with parallel sets of one-dimensional elements replaced by the continuous surface. Typical node displacements and forces in a grid plane as a beam in local coordinates is simplified with 3 degree of freedom (DOF) at each node as shown in Figure 2.

**Figure 2. The idealized discrete system a) the replaced discreate system b) The individual one dimensional element with 3 degree of freedom at each node.**




The degrees of freedom of the element are the local torsion, bending rotation and translation at each end. Since the angular displacements are obtained from the pure torsion member, the torsional DOF's are independent

## II. PROBLEM FORMULATION

The usual approach in formulating problems of beams, plates, and shells is based on deformations in the corresponding differential equation of the beam, plate, or shell. In case action of transverse loads the governing differential equation of the plates can be obtained as;

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)=q(x,y)$$

where w=w(x,y) is the transverse deflection of the plate, q(x,y) is the external loads, D is flexural rigidity of plate. In engineering practice, for a wide variety of plate problems it is necessary to describe the governing equation of motion of plates in a general mathematical form. One dimensional element can replace the continuous two dimensional plates by the appropriate way, the inertia forces due to the lateral translations into the governing differential equation for static equilibrium. By representing the plate with assemblage of individual beam elements interconnected at their neighboring joints, the system cannot truly be equal to the continuous structure, however sufficient accuracy can be obtained similar to the static case (Karasin, 2015; Karasin et al. 2015). Therefore plates can be modeled as an assemblage of individual beam elements interconnected at their intersecting joints.

## III. CASE STUDY

The finite grid method and the other solution methods for maximum deflection values of thin rectangular plates with variation of thickness were compared as case study. The results were compared with the analytical solutions. The error percentage for maximum deflection values of 8 different types of samples in the literature were calculated with various loading and boundary conditions taking into account. An algorithm developed in Matlab using the stiffness and load matrices to determine Finite grid method solution (Matlab, 2015). The finite element method was calculated using Sap2000 and Ansys Workbench package programs (Sap2000, 2016; Ansys Workbench, 2013), and the exact solutions were calculated using the data presented (Timoshenko, 1959). In this study, since excact values of Navier solution is well known in the literature, uniformly distributed load condition of rectangular plates with all edges simple supported cases were considered. The number of plate subdivisions and plate thickness cause variation of the error percentage for maximum deflection values. The variations of the error percentage for simple supported rectangular plates are illustrated in Figs.3-4. On the other hand the error rates for the two different thickness cases are given in Figs.5-6.

**Figure*3*. Variation of Error percentage for *v*=0.3 and the plate divided by 10x10 division in each x- and y-directions**

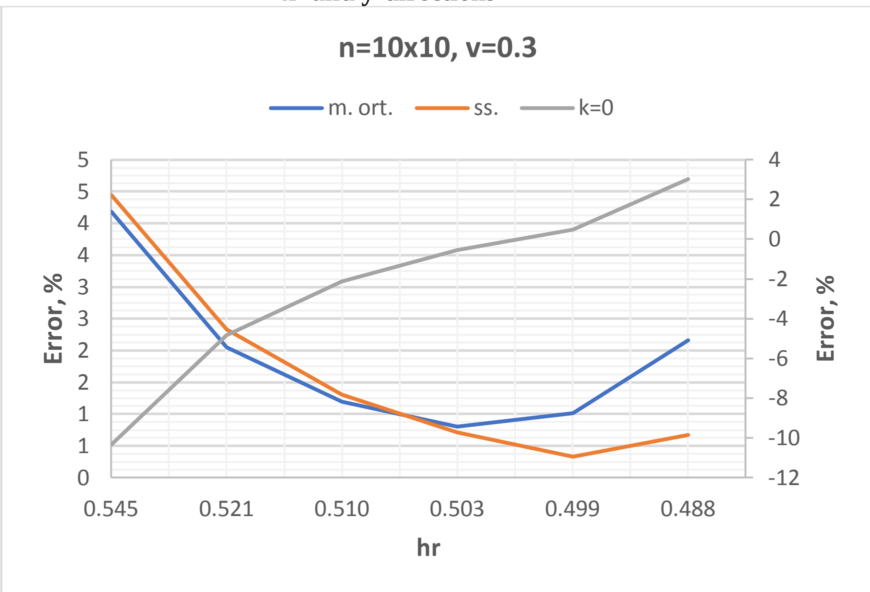



**Figure 4. Variation of Error percentage for *v*=0.3 and the plate divided by 16x16 division in each x- and y-directions**

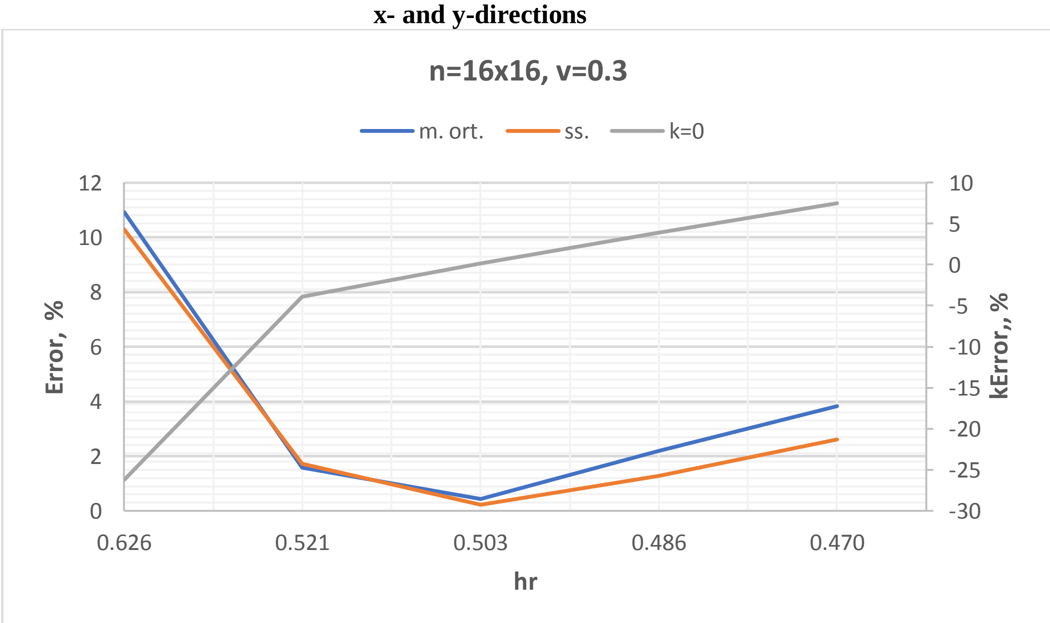


**Figure 5. Variation of Error percentage for *v*=0.3 and the plate thickness of h= 0.05 units**

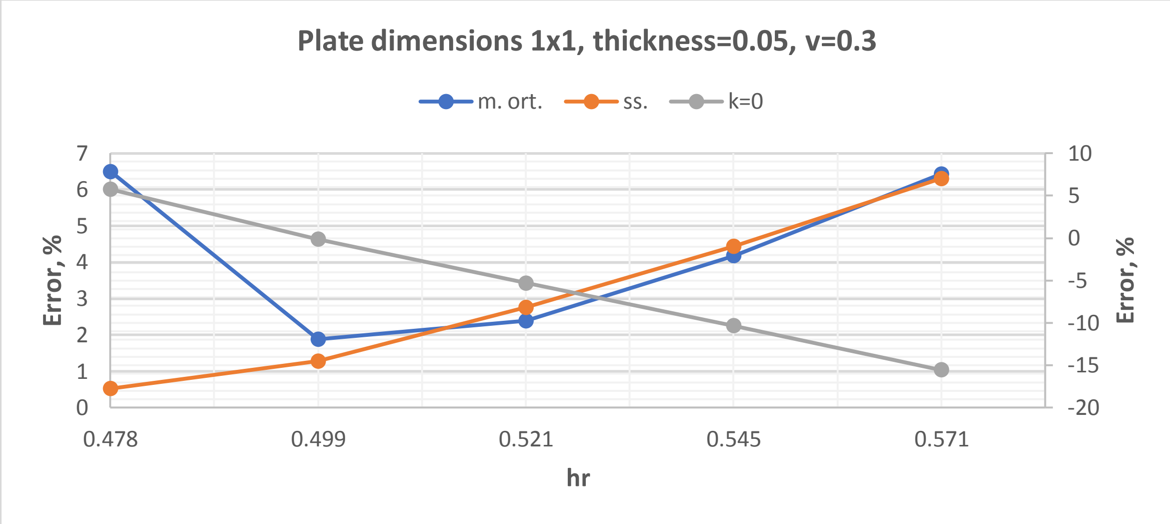
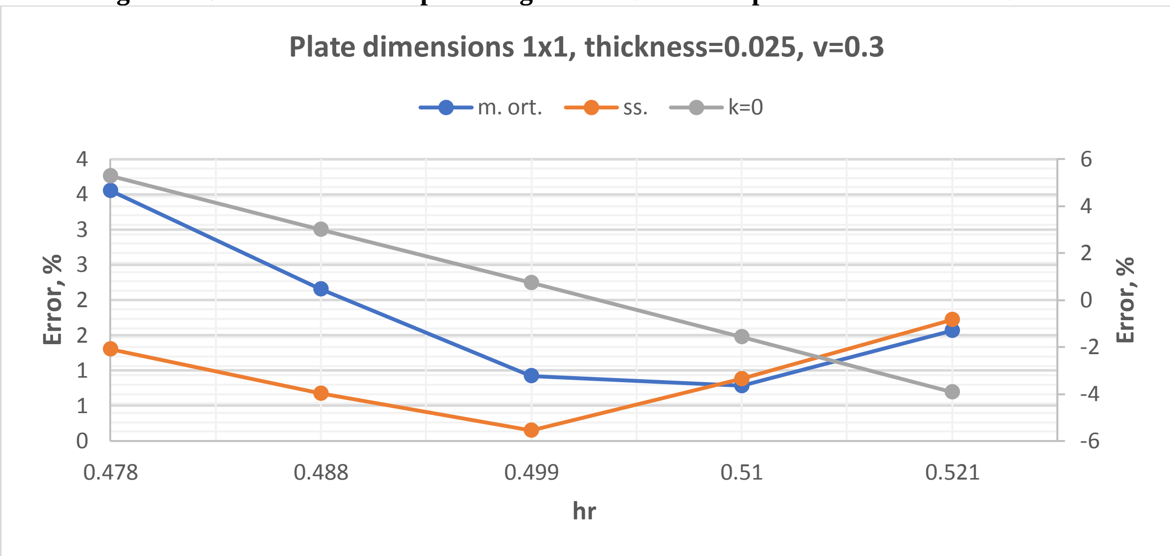



**Figure 6. Variation of Error percentage for *v*=0.3 and the plate thickness of h= 0.025 units**




In case of the plate thickness of h= 0.025 m and h=005 with the plate sizes of 1x1 m. for b / a = 1, plate poisson ratio *v*= 0.3 and the flexural rigidity of the plate, D is 1 considered to analysis the cases. As illustrated in Figs. 5-6. As a result of the analysis it can be noted that the variation of error percentages for normalized maximum deformation values shown sensitivity with respect to plate thickness and plate subdivisions as shown in the figures.

The normalized maximum deflection values with the reference results solved by analytical method shown the average error rates calculated. According to the analysis made, as the number of mesh size increases the error rate in finite grid method also partially increases. On the other hand as plate thickness increases the error percentage obviously decreases. As error rate taken into consideration the solution method provides practical and modeling facilities for thin plate solutions.

## IV. CONCLUSIONS

This method as a grid work analogy called the Finite Grid Solution involving discretized plate properties mapped onto equivalent beams with adjusted parameters and matrix displacement analysis are used to develop a more general simplified numerical approach. The one dimensional element generation using any convenient numbering scheme collected all displacements for each nodal point in a convenient sequence the stiffness and of the system for any type of grids. Then the finite element based matrix methods is used to determine stiffness and geometric stiffness matrices of one-dimensional beam elements by exact shape functions. These individual element matrices are used to form the system load and stiffness matrices of two dimensional plates. It can be concluded that the finite grid method as an approximate numerical method has an acceptable error rate considering that it provides practical and modeling facilities for thin plate solutions. However the analysis performed shows sensitivities in error percentaces with respect to thicknesss and number of subdivisions for rectangular thin plates. The variation of the error percentaces implies that number of subdivisions and the thicknesses should be considered for the best solution.

**Conflict of interest**
There is no conflict to disclose.

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